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MAXIMUM LIKELIHOOD PARAMETER BASED ESTIMATION FOR IN-DEPTH PROGNOSIS INVESTIGATION OF STOCHASTIC ELECTRIC FIELD STRENGTH DATA

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ABSTRACT
This work centers on parametised investigation of the field strength fading characteristics of microcellular wireless LTE channels operating in typical urban terrains. Due to the blockades between the transmitting NodeBs and the mobile station receiver, the fading characteristics of LTE channel are highly stochastic with respect to time or transmitting distance and possesses their own distinctive property. To robustly investigate the field strength fading characteristics, detail drive test measurement has been conducted in five different site locations in waterline area of Port Harcourt City Nigerian at 2.6GHz. From the measured field data, the parametised amplitude of the measured field data is estimated by using the Maximum Likelihood Estimation (MLE) based on different probability distribution functions. The results of the parametised MLE estimate for the acquired field data has been shown, analyzed and reported for each studied location. It is supposed that results of this research this work would serve as a first-hand information for effective communication system design and deployment of future cellular broadband mobile network in similar radio signal propagation terrains.

KEYWORDS: Fading, Channel, Field strength, Maximum likelihood estimate, Probability distribution function**,** Broadband networks

INTRODUCTION

In cellular broadband communication systems, the natural and man-made obstructions between the transmitting and mobile station antenna largely influence the received field strength of the propagated signal power, thus leading to fluctuating and degraded system quality of service. An accepted criterion of performance monitoring and optimization such communication systems is to have detail knowledge about stochastic field signal statistics the channels for re-planning of its networks. Also, according to Isabona *et al*. (2013) and Isabona and Konyeha, (2013), quantified and parametised understanding of the propagated field strength behaviour in the radio links is required to carry out in-depth prognosis analysis of the level of signal coverage fluctuations in the cellular broadband communication networks (Isabona and Obahiagbon, 2014).

In the past years, scholars have explored a number of approaches to study stochastic signal fading phenomenon. For example, in Xiao-Li *et al*. (2018), transmit power estimation centered on Signal Strength of wireless network with cooperative receiver nodes is presented using Maximum Likelihood (ML) estimation technique. Their results reveals that numerical experiments validate the explored ML theoretical discoveries. In Nikola *et al*. (2005), the authors explored maximum likelihood estimation method combined with signal statistics is explored to determine the intensity modulated fiber optic links.

Abiodun and Ojo, (2019), worked on realistic predictive modelling of stochastic path attenuation losses in wireless channels over microcellular urban, suburban and rural terrains using probability distribution functions. The results of their study revealed the normal distribution was most suitable for the statistical predictive modelling signal path loss data. Similar predictive analysis and reports in (Krishnamoorthy, 2006; Salo *et al*., 2005; Fengyu *et al*., 2005).

This work centres on parametised investigation of the field strength fading characteristics of microcellular wireless LTE channels operating in typical urban terrains. To robustly investigate the field strength fading characteristics, detail drive test measurement is piloted in five different locations in waterline area of Port Harcourt City Nigerian at 2.6GHz. From the measured field data, the parametised amplitude of the measured field data is estimated by using the Maximum Likelihood Estimation (MLE) based on different probability distribution functions. The results of parametised MLE estimate of the acquired field data fading characteristics has been revealed, analyzed and reported for each studied location.

THEORETICAL FRAMEWORK

Maximum Likelihood Estimate and Statistical Probability Models

Maximum likelihood estimation (MLE) is a technique of estimating to obtain the parameters of a statistical model, given the observations (i.e. the observed data). This statistical model contains the unknown parameters. Those values of the parameters that maximize the probability of the observed data are referred to as the maximum likelihood estimates. The likelihood function or model is the probability density function (PDF) of the particular observations, and the MLE solution is the parameter that maximizes this joint PDF.

In communication theory, though there exist a number of PDF models, but the problem is in choosing the right one for effective prognosis analysis a particular datasets. In this work, the concentration is on Normal, lognormal, Nakagami, Rician, Weibull and Rayleigh PDFs.

(a) Normal Distribution Model

The normal distribution possess key two distribution parameters. The first parameter is tagged the mean (μ), and the second one is called the standard deviation (σ) or the variance (σ^2). The normal PDF and PDF can be determine using (Isabona and Konyeha, 2015; Krishnamoorthy, 2006):

$$f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] \quad (1)$$

$$F(x,\mu,\sigma) = \frac{1}{2}\left[1 + erf\left(\frac{(x-\mu)}{\sigma\sqrt{2}}\right)\right] \quad (2)$$

The maximum likelihood estimators for the normal distribution are the μ and σ; they can be obtained using the expression in (3) and (4):

$$\mu = \frac{1}{N}\sum_{i=1}^{N}(x_i) \quad (3)$$

$$\sigma = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu_i)^2 \quad (4)$$

where x and N indicate the measured sample and the measurement sample number.

(a) Lognormal Distribution Model

The lognormal distribution, also generally termed Galton or Gaussian distribution, is applicable the desired quantity of interest must be positive. The lognormal PDF and CDF can be defined by (Krishnamoorthy, 2006):

$$f(x,\mu,\sigma) = \frac{1}{x\omega\sqrt{2\pi}}\exp\left[\frac{-(\ln x-\mu)^2}{2\omega^2}\right] \quad (5)$$

$$F(x,\mu,\sigma) = \frac{1}{2}+\frac{1}{2}erf\left[\frac{-(\ln x-\mu)^2}{\sqrt{2}\omega}\right] \quad (6)$$

In (7), μ and ω maximum likelihood estimators and they represent the shape and scale distribution parameters for the lognormal. The mean and standard deviation for lognormal can be expressed as:

$$\mu = \exp\left(\mu + \frac{\omega^2}{2}\right) \quad (7)$$

$$\sigma = \exp\left(2\mu + \omega^2\right)[\exp\left(\omega^2\right) - 1] \quad (8)$$

where x stands for the measured sample.

(b) Weibull Distribution Model

The weibull distribution, which generally employed for reliability analysis, make use of λ, and c as its shape and scale (Weibull slope) distribution parameters. The Weibull PDF and CDF can be defined by (Krishnamoorthy, 2006, Abiodun and Ojo, 2019):

$$f(x,\lambda,c) = \frac{c}{\lambda}\left(\frac{x}{\lambda}\right)^{c-1}\exp\left[-\left(\frac{x}{\lambda}\right)^{c}\right] \quad (9)$$

$$F(x,\lambda,c) = 1-\frac{c}{\lambda}\left(\frac{x}{\lambda}\right)^{c-1}\exp\left[-\left(\frac{x}{\lambda}\right)c\right] \quad (10)$$

The distribution parameters, μ and σ can be obtained the expressions in (11) and (12) respectively:

$$\mu = \lambda\Gamma\left(1+\frac{1}{c}\right) \quad (11)$$

$$\sigma = \sqrt{\lambda^2\Gamma\left(1+\frac{2}{c}\right)-\left[\Gamma\left(1+\frac{1}{c}\right)\right]^2} \quad (12)$$

where x stands for the measured sample.

(c) Rayleigh Distribution Model

The Rayleigh distribution is a continuous probability distribution and also a special (singular) case of the Weibull distribution. The Rayleigh PDF and CDF are given by (Krishnamoorthy, 2006, Abiodun and Ojo, 2019):

$$f(x,\sigma) = \frac{x}{\sigma}\exp\left[-\left(\frac{-x^2}{2\sigma^2}\right)\right] \quad (13)$$

$$F(x,\sigma) = 1-\exp\left[-\left(\frac{-x^2}{2\sigma^2}\right)\right] \quad (14)$$

The distribution parameters, μ and σ_m can be obtained the expressions in (15) and (16) respectively:

$$\mu = \sigma\sqrt{\frac{\pi}{2}} \quad (15)$$

$$\sigma_m = \sigma\sqrt{\frac{4-\pi}{2}} \quad (16)$$

where x stands for the measured sample.

(d) Nakagami Distribution Model

The Nakagami distribution, also termed Nakagami-*m* distribution, behave roughly and evenly near its mean value. The Nakagami PDF and CDF can expressed as (Krishnamoorthy, 2006):

$$f(x,m,a)=\frac{2m^m}{\Gamma(m)a^m}x^{2m-1}\exp\left[-\frac{m}{a}x^2\right] \quad (17)$$

$$F(x,m,\Omega)=\frac{Y\left(m,\frac{m}{a}x^2\right)}{\Gamma(m)} \quad (18)$$

In (20), μ and ω represent the shape and scale distribution parameters for the Nakagami. The mean and standard deviation for Nakagami can be expressed as:

$$\mu=\frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m)}\left(\frac{a}{m}\right)^{1/2} \quad (19)$$

$$\sigma=\sqrt{a\left(1-\frac{1}{m}\left(\frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m)}\right)^2\right)} \quad (20)$$

where x stands for the measured sample.

(e) Rician Distribution

In communication, the Rician distributions model are usually employed to study stronger line-of-sight fading channels. The Rician PDF and CDF can expressed as (Krishnamoorthy, 2006):

$$f(x,v,\sigma)=\frac{x}{\sigma^2}\exp\left[\left(\frac{-x^2+v^2}{2\sigma^2}\right)I_o\left(\frac{xv}{\sigma^2}\right)\right] \quad (21)$$

$$F(x,v,\sigma)=1-Q_1\left(\frac{v}{\sigma},\frac{x}{\sigma}\right) \quad (22)$$

In (24), μ and ω represent the shape and scale distribution parameters for the Rician. The mean and standard deviation for Rician can be expressed as:

$$\mu=\sigma\sqrt{\frac{\pi}{2}}L_{1/2}\left(\frac{-v^2}{2\sigma^2}\right) \quad (23)$$

$$\sigma=2\sigma^2+v^2-\frac{\pi\sigma^2}{2}L_{1/2}^2\left(\frac{-v^2}{2\sigma^2}\right) \quad (24)$$

where x stands for the measured sample. $I_o(z)$ and $Q_1(z)$ represent the modified Bessel function and Marcum Q function, respectively.

METHODOLOGY

Data Collection

The measurement campaign has been performed around five operational Long term Evolution (LTE) cellular networks base station (BS) sites in Waterline areas of Port Harcourt City, with concentration on built-up busy urban streets, and roads. It is a typical urban area with a flat topography and mixed commercial and residential building edifices. As revealed in table 1, the BS antenna heights range from 28m to 45m, elevated above the ground level to broadcast signals in three sectors configuration. With the aid of drive test equipment which include the Global Positioning System (GPS), HP Laptop, two Samsung Galaxy mobile Handsets (Model-SY 4) and network scanner, signal power measurements were conducted along different routes round the cell sites, in active mode. Specifically, drive tests around sites I, II and IV were performed via non-line of sight (NLOS) routes, while that of sight III was piloted through line of sight (LOS) route, such that there were no obstructions between the BS and user equipment terminal. A snap shot of data collection in route I is revealed in figure 3. All the test equipment were connected together with USB cables and housed in a Gulf car before the field drive test measurement. Also, both the Samsung handsets and the HP laptop were both enhanced with Telephone Mobile Software (TEMS, 15.1 version), which enable us to access, acquire and extract signal power data, including serving BS information after measurement. A total of 1,502 signal power data points were extracted for further analysis using MapInfo and Microsoft Excel spreadsheet.

The measured signal power, which is called RSRP, is related to electric field strength, E_{FS} by (Isabona, *et al,* 2013):

$$E_{FS}(dB\mu/m) = RSRP(dBm) - G + A \quad (25)$$

$$A = 10\log\left(\frac{1.64\lambda}{4\pi}\right) + 145.8 \quad (26)$$

where, G is the antenna gain in dB, λ is the signal transmitting wavelength in m and RSRP indicates Reference Signal Receive Power.

Table 1: Measurement Campaign Parameters in LTE Network

	Site I	Site II	Site III	Site IV	Site V
Parameter			Value		
Operating Frequency (MHz)	2600	2600	2600	2600	2600
BS Antenna Height (m)	28	30	45	32	38
BS antenna gain (dBi)	17.5	17.5	17.5	17.5	17.5
Transmit power (dB)	43	43	43	43	43
Feeder Loss (dB)	3	3	3	3	3
Transmitter cable loss (dB)	0.5	0.5	0.5	0.5	0.5
Mobile antenna height (dB)	1.5	1.5	1.5	1.5	1.5

RESULTS AND DISCUSSION

Displayed in figures 1 to 5 are acquired stochastic electric field strength data via drive test over the period of measurement in each study terrain sites. Their stochastic fading distributions characteristics are shown in Figure 6 (a-f) using the six probability distribution models (i.e. Normal, lognormal, Nakagami, Rician, Weibull and Rayleigh distributions). Figure 7 (a-f) reveals the corresponding cumulative distribution functions (CDFs) model fittings connected to the pdfs depicted in Figure 6(a-f). A fluctuating fading pattern of measured field strength along the measurement locations can been seen in all the Figures of 1 to 5 and this is roughly constant across the study location sites. The scenario can be attributed to the uneven proliferation of natural and man-made obstructions between the transmitting and mobile station antenna.

More importantly, to ascertain how accurately each statistical distribution fits with the measured electric field strength data, the estimated Maximum Likelihood Parameter Estimation method were considered as benchmark**.** Also, to reveal how well the investigated distribution function models fit into the stochastic fading behaviour of the measured electric field strength data, MLE statistics were further explored and the resultant estimated values using MLE are revealed in table 2. From the table, the lognormal distribution model fitting display the best maximum log likelihood values of -312 and 415 in locations 1 and 2. It is an indication that lognormal distribution model is a better fit to the measured electric field strength data in the two locations compared to other distributive models. As revealed in from table 2, while Nakagami model gave the optimal maximum log likelihood estimate in location 3, Normal and Rician models provided the best maximum log likelihood estimates of -200 and 210 in locations 4 and 5, respectively. Unlike the results in (Abiodun and Ojo, 2019), different distributive fitting results on the measured electric field strength data in the respective locations is revealed in this work; it simply shows that one particular distributive model cannot be generalized as a fitting model to the study locations. Such performances may be attributed to differences that exist in building height structures, roof tops, street/roads widths around the five study location sites. In general, the graphs in figure 2 (a-f) clearly shows that Rayleigh and Weibull distribution displayed the poorest fits to empirical electric field strength. This simply implies that there are no multiple paths of condensed scattered electric field signals reaching a receiver along the measurement routes in the study location sites

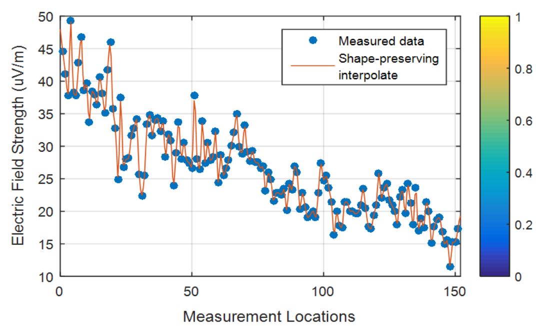

Fig. 1: Stochastic electric field strength data in Site location I

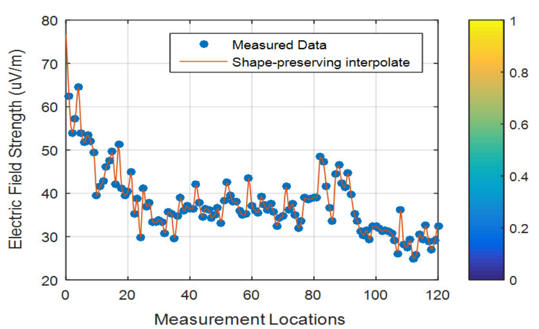

Fig. 2: Stochastic electric field strength data in Site location II

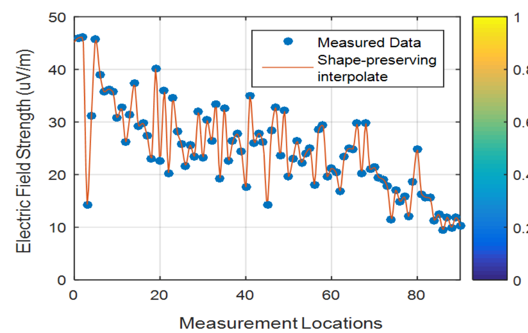

Fig. 3: Stochastic electric field strength data in Site location III

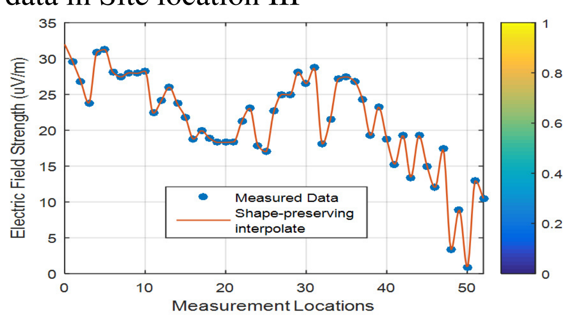

Fig. 4: Stochastic electric field strength data in Site location IV

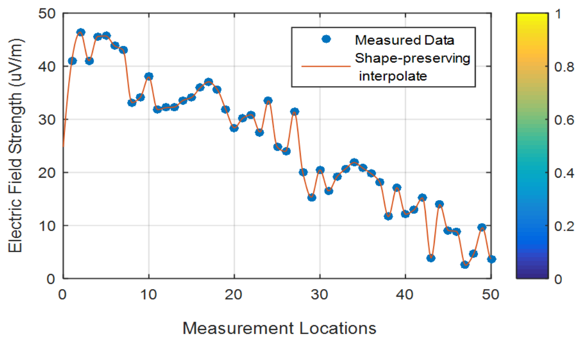

Fig. 5: Stochastic electric field strength data in Site location V

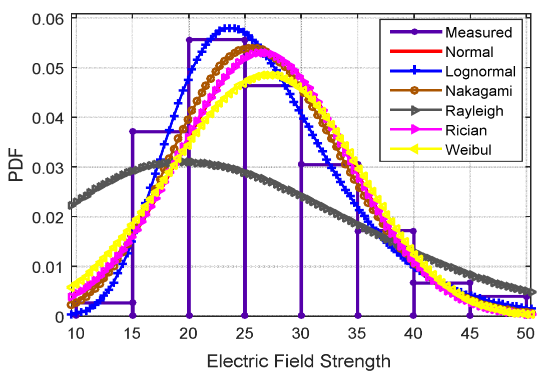

Fig. 6 (a): Stochastic electric field strength data with different PDFs fitted plots in Site location I

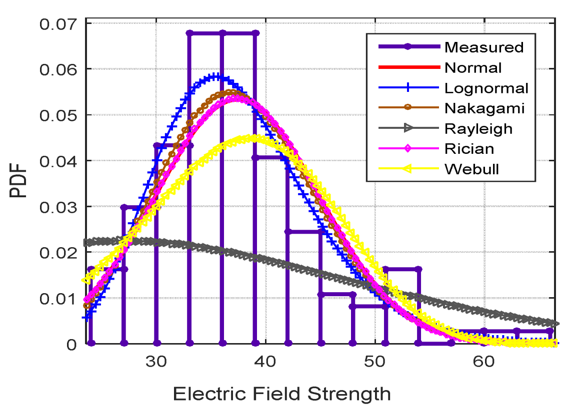

Fig. 6 (b): Stochastic electric field strength data with different PDFs fitted plots in Site location II

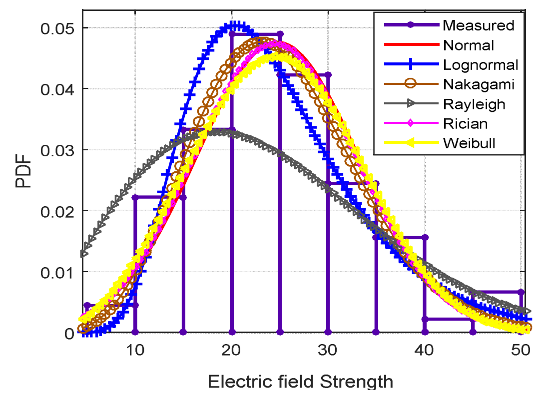

Fig. 6 (c): Stochastic electric field strength data with different PDFs fitted plots in Site location III

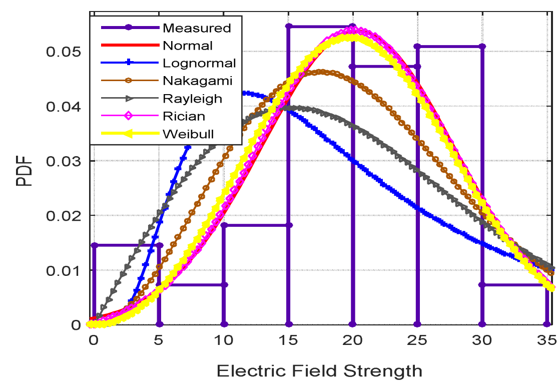

Fig. 6 (d): Stochastic electric field strength data with different PDFs fitted plots in Site location IV

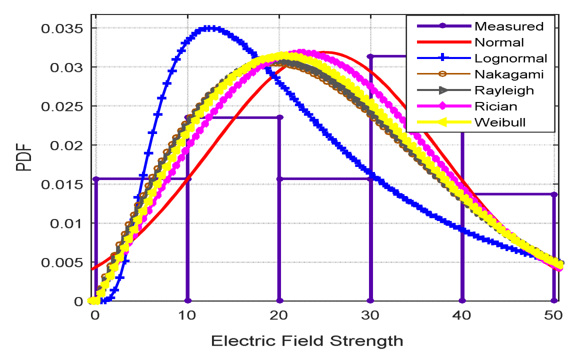

Fig. 6 (f): Stochastic electric field strength data with different PDFs fitted plots in Site location V

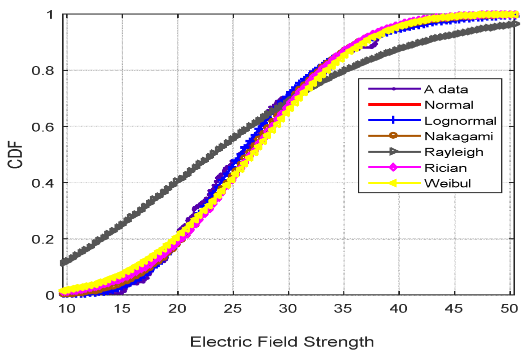

Fig. 7 (a): Stochastic electric field strength data with different CDFs fitted plots in Site location I

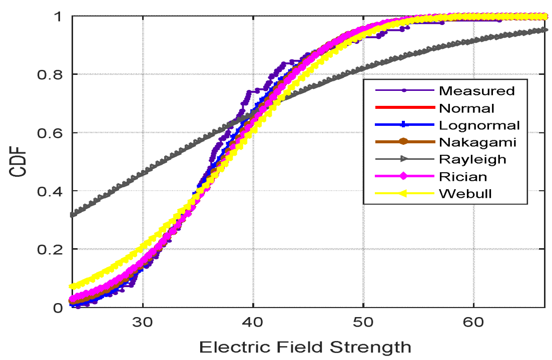

Fig. 7 (b): Stochastic electric field strength data with different CDFs fitted plots in Site location I

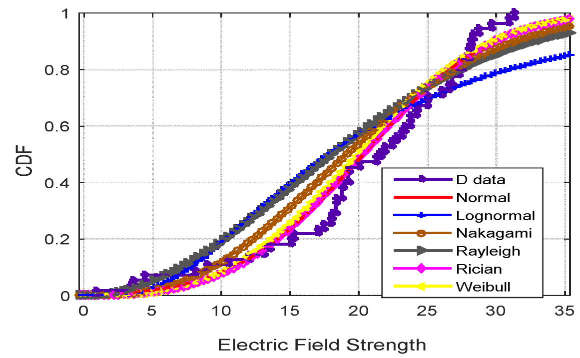

Fig. 7 (d): Stochastic electric field strength data with different CDFs fitted plots in Site location IV

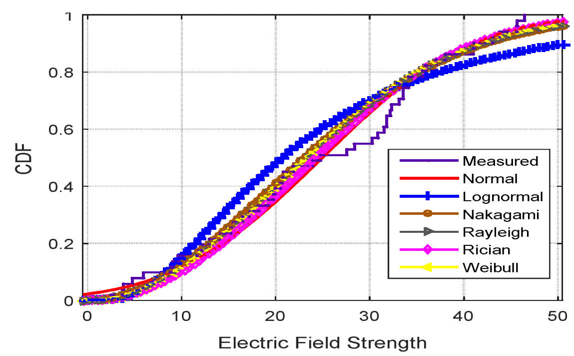

Fig. 7 (f): Stochastic electric field strength data with different CDFs fitted plots in Site location V

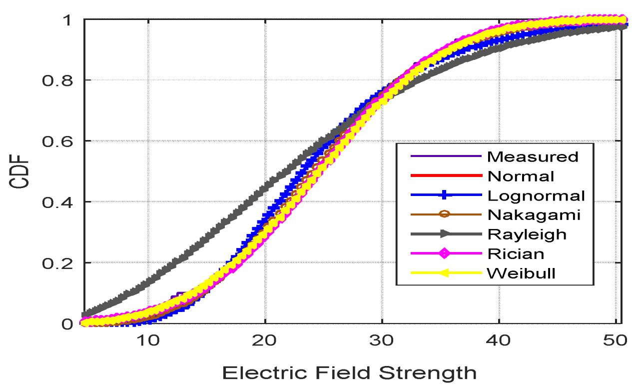

Fig. 7 (c): Stochastic electric field strength data with different CDFs fitted plots in Site location III

Table 2: Statistical Maximum Likelihood Estimation Parameters and Values

		Location				
Distribution function	MLE Parameters	1	2	3	4	5
Normal	Log likelihood Estimate	-518	-421	-318	-187	-200
	Mean	26.63	37.41	24.70	20.33	24.92
	Variance	56.57	55.97	70.19	55.14	156.40
Lognormal	Log likelihood Estimate	-510	-412	-319	-212	-208
	Mean	26.65	37.39	24.82	22.08	26.59
	Variance	57.56	51.22	87.51	259.61	463.95
Nakagami	Log likelihood Estimate	-514	-417	-316	-194	-200
	Mean	26.68	37.44	24.71	19.85	24.58
	Variance	53.90	52.67	69.00	73.08	170.20
Rayleigh	Log likelihood Estimate	-554	-490	-331	-196.50	-200
	Mean	24.52	33.80	23.10	19.16	24.66
	Variance	164.35	312.27	145.90	100.32	166.23
Rician	Log likelihood Estimate	-517	-421.39	-318	-188	-199
	Mean	26.64	37.41	24.70	20.35	25.11
	Variance	56.13	55.51	69.40	53.24	144.06
Weibull	Log likelihood Estimate	-520	-431	-317.80	-191.24	-200
	Mean	26.60	37.13	27.59	20.09	24.82
	Variance	62.97	77.13	71.39	52.71	156.80

CONCLUSION

Through the use of the detailed maximum likelihood estimator in correspondent with different stochastic distribution models, the statistical fading characteristic of measured electric field strength data acquired in five uneven microcellular urban terrains of deployed LTE broadband communication channels has been investigated and analyzed in this research work. From the results summary, the lognormal distribution model fitting display the maximum log likelihood values of -510 and -412 in locations 1 and 2. It is an indication that lognormal distribution model is better fit the measured electric field strength data in the two locations compared to other distributive models. Also from the results, while Nakagami model gave the maximum log likelihood estimate of -316 in location 3, Normal and Rician models provided the maximum log likelihood estimates of -194 and -199 in locations 4 and 5, respectively. These different distributive fitting results on the measured electric field strength data in the respective locations clearly reveals that one particular model cannot be generalized as a fitting function terrain. It is also mainly caused by differences that exist in building height structures, roof tops, street/roads widths in the five study locations

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