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THE COMBINATORIAL GENERATION AND THE MATRIX ELIMINATION MODEL FOR THE LONG-RUN DIAGNOSTIC STABILITY OF AN AUTOREGRESSIVE DISTRIBUTED LAG (ARDL) MODEL

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ABSTRACT
This study examines the long – run stability of an Autoregressive Distributed Lag (ARDL) model with intercept and no constant trend, using quarterly data from CBN annual statistical bulletin that span from 1999Q1 to 2011Q2 and 2000Q1 to 2012Q4. The parameters of the long – run stability of ARDL model are not all statistically significant in their respective p – values and as such cannot be totally relied upon for statistical estimation. Then, we proposed a matrix elimination model which is motivated by the p – values of the respective parameters to systematically eliminate at a time an exogenous variable and its corresponding parameter whose p – value is less significant from the system. Using econometric package (EView), it was investigated that there is collaboration in the long – run stability between the ARDL model and the Binomial coefficient model for different order of n(K). The findings show that the system (model) now accommodates parameters whose p – values are all statistically significant at 5% level of significance. This now paves way for an extended understanding for decision and policymakers to formulate a mechanism to maintain long – run stability in Foreign Reserves.

KEYWORDS: *Binomial coefficients, Econometric View, Diagnostic Stability, Minimum information Criteria, Parsimonious Variables, Over-Parameterization*

Introduction

This paper considers an Autoregressive distributed lag model (ARDL) which is also known as the bound testing approach developed by Pesaran *et al*. (2001) to estimate long – run and short – run parameters in a single model. In this study, emphasis is on the long – run stability of foreign reserves in Nigeria and the binomial coefficients model to systematically examine the diagnostic stability for different order of n(k) of an ARDL model. Using econometric view package, we shall establish the long – run stability of an ARDL model on the sequence of models of the binomial coefficients. The result shall determine which family has its model least parsimonious. Then slight modification in the choice of exogenous variables as considered by Irefin and Yaaba (2013) with the inclusion of second lag in reserves, third lag in Gross domestic product, trade openness and first lag in foreign debt shall be investigated. Accordingly Irefin and Yaaba (2013) were able to ascertain that income (GDP) is the major determinant of foreign reserves holding in Nigeria.

Ojo (2013) was able to analyze between the full and subset autoregressive polynomial distributed lag models without intercept, where both exogenous and dependent variables are stationary. He developed a scheme to eliminate irrelevant lags as a build – up for the subset model. He later discovered that the subset model perform better using information criteria (Akaike information criterion and Schwarz criterion) and residual variance.

Yi-Yi (2010) as cited in Ojo (2013) examined the inclusion and the consequence of lag variables in an autoregressive distributed lag model for dependent variable and for several exogenous variables. The level of outcome on policy variables are determined by time lags respectively.

Bankole and Shuaibu (2013) applied vector autoregressive model and ascertain that a drop in income generated from oil price has gross effect in reserves in the long – run with a minimal effect on reserves in the short – run. This again established the fact that gross domestic product is a major determinant in reserve holding in Nigeria.

Abdullateef and Waheed (2010) explored combination of ordinary least squares and vector error correction methods. It was observed that a variation in reserves has no influence on domestic investment and inflation rate, but it stimulates foreign direct investment and exchange rates. The finding stipulates the need for broader reserve management targeted in maximizing the advantage derived from oil exportation to enhance internal investment.

Atif *et al.* (2010) explored the influence of financial development and trade openness on gross domestic product exhausting autoregressive distributed lag approach formulated by Pesaran *et al.* (2001). The findings show that, trade openness and financial development induced economic growth.

The study carried by Shahbaz and Faridul (2011) adopted autoregressive distributed lag (ARDL) model to estimate the coefficients of the long run relationship and the error correction model. In their investigation, it was found that financial development reduces income differences and it is aggravated by financial instability.

Iwueze *et al.* (2013) discussed how levels and trend can influence foreign reserves in Nigeria. They explored data from annual CBN statistical bulletin 1999 – 2008 using ARIMA model. Their findings showed that there is need for data logarithmic transformation for variance stability and make the distribution normal. They claim that the autoregressive – integrated moving average for order (2, 1, 0) best disrobes the pattern in the transformed data.

Charles – Anyaogu (2012) analyzed data collected from CBN statistical bulletin that span from 1980 – 2009 using vector autoregressive model and Wald test, it was found that the exogenous variable (GDP) is significant in explaining foreign reserves. The outcome is different with difference in time lag in reserves and suggested that an enabling environment should be built for trade openness to increase GDP for Nigeria economy.

Ajayi and Oke (2012) investigated the burden external debt has on the Nigerian development and economy. They implemented ordinary least squares exploring data from CBN on variables such as national income, debt service payment, external reserves and interest rate among others. They found that high level of external debt on nation income and per capital income of the nation can lead to currency devaluation, retrenchment, continuous industrial strike and failure in educational system.

The Buffer stock model of Frenkel and Jovanovic (1981) defined reserve movements in continuous time period as a Weiner process given as:

$$dR(t) = -\mu dt + \sigma dW(t) \quad (1)$$

where: $R_t =$ reserves held in time t
W_t = Standard Weiner process with zero mean and variance t
μ = Deterministic part of the instantaneous change in reserves
σ = Standard deviation of the Weiner increment in reserves
R^* = denotes the optimal stock of reserves
Then, $R(t)$ is characterized by

$$R(t) = R^* - \mu t + \sigma W(t) \quad (2)$$

Autoregressive Distributed Lag (ARDL) model by Pesaran *et al*. (2001) was developed to estimate Frenkel and Jovanovic's buffer stock model, but with a slight modification (Irefin and Yaaba (2013) The ARDL $(p, q_1, q_2, \ldots, q_k)$ model following Pesaran *et al*. (2001) can be written as follows:

$$\emptyset(L,p)y_t = \sum_{i=1}^{k} \beta_i\,(L,q_i)x_{it} + \delta' w_t + u_t, i = location\ at\ time\ t;\ t = 1,2,\ldots,n;\ i = 1,2,\ldots,k \quad (3)$$

Where

$$\emptyset(L,p) = 1 - \emptyset_1 L - \emptyset_2 L^2 - \ldots \emptyset_p L^p \quad (4)$$

and

$$\beta_i(L,q_i) = 1 - \beta_{i1}L - \beta_{i2}L^2 - \cdots - \beta_{iqi}L^{qi},\ \ i = 1,2,\ldots,k \quad (5)$$

From equation (3), $y_t$is the dependent variable, x_{it} denotes the independent variables, L is the lag operator, and w_t is the $s \times 1$ vector of deterministic variables, including intercept terms, dummy variables, time trends and other exogenous variables as cited in Mosayeb *et al*. (2005).
According to Pesaran and Pesaran (2001), as cited in Wilson and Chaudhri (2004), the long-run coefficient can be estimated by:

$$\beta_i = \frac{\hat{\beta}_i(l,\hat{q_i})}{\Omega(l,\hat{p})} = \frac{\hat{\beta}_{i0}+\hat{\beta}_{i1}+\cdots+\hat{\beta}_{qi}}{1-\hat{\emptyset}_1-\hat{\emptyset}_2-\cdots-\hat{\emptyset}_p} \qquad \forall\ i = 1,2,\ldots,k \quad (6)$$

$$y_t - \hat{\theta}_0 - \hat{\theta}_1 x_{1t} - \hat{\theta}_2 x_{2t} - \cdots - \hat{\theta}_k x_{kt} = \varepsilon_t \qquad \forall\ t = 1,2,\ldots,n \quad (7)$$

where $i = location\ at\ time\ t.$
(Pesaran and Pesaran (2001)).

In Equation (7) the constant term is equal to:

$$\hat{\theta}_0 = \frac{\hat{\beta}_0}{1-\hat{\emptyset}_1-\hat{\emptyset}_2-\cdots-\hat{\emptyset}_p}$$

The Error correction model of the selected ARDL model can be obtained by rewriting Equation (3) in terms of lagged levels, first difference and w_t as follows:

$$\Delta y_t = -\emptyset(L,\hat{p})ECM_{t-1} + \sum_{i=1}^{k}\beta_{io}\,\Delta x_{1t} + \delta'\Delta w_t - \sum_{j=1}^{\hat{p}-1}\varphi^*\,y_{t-j} - \sum_{i=1}^{k}\sum_{j=1}^{\hat{q}_i-1}\beta_{ij}^*\,\Delta x_{i,t-j} + u_t \quad (8)$$

where, the error correction term is defined by

$$ECM_{t-1} = y_t - \sum_{i=1}^{k}\hat{\theta}_i\,x_{it} - \omega' w_t \quad (9)$$

METHODOLOGY

Modified ARDL Model

Consider the function:

$$R_t = \alpha G_t^{\alpha_1} T_t^{\alpha_2} M_t^{\alpha_3} E_t^{\alpha_4} S_t^{\alpha_5} \varepsilon_t \quad (10)$$

where $R_t, G_t, T_t, M_t, E_t, S_t$ are the Foreign Reserve, Gross Domestic Product, Trade Openness, Monetary Policy Rate, Exchange Rate and Foreign Debt at time t respectively and $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are parameters of the respective exogenous variables

ASSUMPTIONS OF ARDL MODEL:

i. The errors ε_t are serially independent with σ^2 , $\varepsilon_t \sim iid\,(0,\sigma^2)$.

ii. The errors are uncorrelated with $\Delta_{t+h}, \forall\, h \in Z$.

iii. $\beta_0 - \Upsilon\alpha_0 = \rho_0, \Upsilon = \vartheta_1, -\Upsilon\alpha_1 = \vartheta_2, -\Upsilon\alpha_2 = \vartheta_3, -\Upsilon\alpha_3 = \vartheta_4, -\Upsilon\alpha_4 = \vartheta_5\ and - \Upsilon\alpha_5 = \vartheta_6$

iv. Maximum lag length (p, n, m, o, q, v) = (2, 3, 0, 0, 0, 1) respectively.

v. $-\beta_3 = a_0' = a_2' - a_1' = -a_3' = -\theta_0' = -\alpha_0' = -\lambda_0' = \mu_0' = -\mu_1' = \vartheta_3 - \theta_0' = \vartheta_4 - \alpha_0' = \vartheta_5 - \lambda_0' = 0.$

vi. $a = b = 1, i = n(K)$

vii. The modified ARDL model must stability in its parameters

viii. $n + 1 = n(k)$

ix. $\frac{n(k)!}{(n(k)-r)!r!} = 0, \forall\, n(k) < r$

x. $\frac{n(k)!}{(n(k)-r)!r!} = 0, \forall\, n(k)r < 0$

xi. $j = r - 1$

The logarithmic linear specification of Equation (10) which is applied in the multivariate cointegration technique is as follows:

$$LogR_t = \alpha_0 + \alpha_1 LogG_t + \alpha_2 LogT_t + \alpha_3 LogM_t + \alpha_4 LogE_t + \alpha_5 LogS_t + \varepsilon_t\,, Log\alpha = \alpha_0 \quad (11)$$

$$r_t = \alpha_0 + \alpha_1 g_t + \alpha_2 t_t + \alpha_3 m_t + \alpha_4 e_t + \alpha_5 s_t + \varepsilon_t \quad (12)$$

Equation (12) represents the simple linear functional formulation of Equation (11).
where $r_t = LogR_t$, $g_t = LogG_t$, $t_t = LogT_t$, $m_t = LogM_t$, $e_t = LogE_t$, $s_t = LogS_t$ and ε_t is the residual term assumed to be normally distributed.

Consider the simple case for compact ARDL as:

$$B(L,p)r_t = \beta_0 + a'(L,n)g_t + \varepsilon_t \quad (13)$$

Equation (13) can be expressed as;

$$r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 r_{t-2} + \ldots + \beta_p r_{t-p} + a_0 g_t + a_1 g_{t-1} + a_2 g_{t-2} + \cdots + a_n g_{t-n} + \varepsilon_t \quad (14)$$

$$r_t = \beta_0 + \sum_{i=1}^{p} \beta_i r_{t-i} + \sum_{i=0}^{n} a_i' g_{t-i} + \varepsilon_t \quad (15)$$

Where L is the lag operator, $B(L,p)$ is the lag polynomial and $a'(L,n)$ is the vector polynomial defined as follows:

$$B(L,p) = 1 - \sum_{i=1}^{p} \beta_i L^i = 1 - \beta_1 L^1 - \beta_2 L^2 - \ldots - \beta_p L^p \quad (16)$$

$$a'(L,n) = \sum_{i=0}^{n} a_i L^i = a_0 + a_1 L^1 + a_2 L^2 + \cdots + a_n L^n \quad (17)$$

By substituting Equations (16) and (17) in Equation (13) we have Equation (18)

$$\left(1 - \sum_{i=1}^{p} \beta_i L^i\right) r_t = \beta_0 + (a_0' + a_1' L^1 + a_2' L^2 + \cdots + a_n' L^n) g_t + \varepsilon_t \quad (18)$$

$$\left(1 - \beta_1 L^1 - \beta_2 L^2 - \ldots - \beta_p L^p\right) r_t = \beta_0 + a'_0 g_t + a'_1 L^1 g_t + a'_2 L^2 g_t + \cdots + a'_n L^n g_t + \varepsilon_t \quad (19)$$

$$r_t - \beta_1 L^1 r_t - \beta_2 L^2 r_t - \ldots - \beta_p L^p r_t = \beta_0 + a'_0 g_t + a'_1 L^1 g_t + a'_2 L^2 g_t + \cdots + a'_n L^n g_t + \varepsilon_t \quad (20)$$

$$r_t - \beta_1 r_{t-1} - \beta_2 r_{t-2} - \ldots - \beta_p r_{t-p} = \beta_0 + a'_0 g_t + a'_1 g_{t-1} + a'_2 g_{t-2} + \cdots + a'_n g_{t-n} + \varepsilon_t \quad (21)$$

$$r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 r_{t-2} + \ldots + \beta_p r_{t-p} + a'_0 g_t + a'_1 g_{t-1} + a'_2 g_{t-2} + \ldots + a'_n g_{t-n} + \varepsilon_t (22)$$

$$r_t = \beta_0 + \sum_{i=1}^{p} \beta_i r_{t-i} + \sum_{i=0}^{n} a_i' g_{t-i} + \varepsilon_t \ (15)$$

Where the coefficients of β_i $(\beta_1, \beta_2, \beta_3, \ldots, \beta_p)$ are parameters of the autoregressive components and p is the lag length of the autoregressive component. While $a_i'(a_0', a_1', a_2', \ldots, a_n')$, $\theta_i'(\theta_0', \theta_1', \theta_2', \ldots, \theta_m')$, $\alpha_i'(\alpha_0', \alpha_1', \alpha_2', \ldots, \alpha_o')$, $\lambda_i'(\lambda_0', \lambda_1', \lambda_2', \ldots, \lambda_q')$ and $\mu_i'(\mu_0', \mu_1', \mu_2', \ldots, \mu_v')$ are the parameters of the polynomial distributed lag component whereas n, m, o, q and v are the lag length of the polynomial distributed lag component. Equation (15) can be extended to more exogenous variable. Thus, the ARDL compact (p, n, m, o, q, and v) model is given by:

$$B(L,p)r_t = \beta_0 + a'(L,n)g_t + \theta'(L,m)t_t + \alpha'(L,o)m_t + \lambda'(L,q)e_t + \mu'(L,v)s_t + \varepsilon_t \quad (16)$$

T

r ... $+$
ε_t (17)

For we which to show that;

$$\sum_{i=1}^{p} \beta_i r_{t-i} = -B(1)r_{t-1} + \sum_{i=1}^{p} \beta_i \Delta r_{t-i} + r_{t-1} \quad (18a)$$

$\sum_{i=0}^{n} a_i'\, g_{t-i} = B(1)\beta' g_t + \sum_{i=1}^{n} a'_i\, \Delta g_{t-i}$ (18b)
$\sum_{i=0}^{m} \theta_i'\, t_{t-i} = B(1)\rho' t_t + \sum_{i=1}^{m} \theta'_i\, \Delta t_{t-i}$ (18c)
$\sum_{i=0}^{o} \alpha_i'\, m_{t-i} = B(1)\delta' m_t + \sum_{i=1}^{o} \alpha'_i\, \Delta m_{t-i}$ (18d)
$\sum_{i=0}^{q} \lambda_i'\, e_{t-i} = B(1)\varphi' d_t + \sum_{i=1}^{q} \lambda'_i\, \Delta e_{t-i}$ (18e)
$\sum_{i=0}^{v} \mu_i'\, s_{t-i} = B(1)\omega' s_t + \sum_{i=1}^{v} \mu'_i\, \Delta s_{t-i}$ (18f)

The purpose of Equation (18a) to (18f) is to formulate the ARDL unrestricted model that enable us estimate the long-run and short-run parameters respectively.

From Equation (16), we have
$B(L) = 1 - \sum_{i=1}^{p} \beta_i L^i = 1 - \beta_1 L^1 - \beta_2 L^2 - \ldots - \beta_p L^p$
Multiplying Equation (16) by $-r_{t-1}$ yields,

$$-B(L)r_{t-1} = -\left(1 - \sum_{i=1}^{p} \beta_i L^i\right) r_{t-1}$$
$$= -(1 - \beta_1 L^1 - \beta_2 L^2 - \ldots - \beta_p L^p) r_{t-1} \quad (19)$$

$$-B(L)r_{t-1} = -r_{t-1} + \beta_1 L^1 r_{t-1} + \beta_2 L^2 r_{t-1} + \ldots + \beta_p L^p r_{t-1} \quad (20)$$

where $L^p r_{t-1} = r_{t-p-1}$ (21)

$$-B(L)r_{t-1} = -r_{t-1} + \beta_1 r_{t-2} + \beta_2 r_{t-3} + \ldots + \beta_p r_{t-p-1} \quad (22)$$

Set $L = 1$

$$-B(1)r_{t-1} = -r_{t-1} + \beta_1 r_{t-2} + \beta_2 r_{t-3} + \ldots + \beta_p r_{t-p-1} \quad (23)$$

where $\sum_{i=1}^{p} \beta_i\, \Delta r_{t-i} = \beta_1 \Delta r_{t-1} + \beta_2 \Delta r_{t-2} + \beta_3 \Delta r_{t-3} + \ldots + \beta_p \Delta r_{t-p}$ (24)

where $\Delta = 1 - L$ from Δr_t and Δ is the first difference of the variables.

$$\sum_{i=1}^{p} \beta_i\, \Delta r_{t-i} = \beta_1(r_{t-1} - r_{t-2}) + \beta_2(r_{t-2} - r_{t-3}) + \ldots + \beta_p\left(r_{t-p} - r_{t-p-1}\right) \quad (25)$$

$$= \beta_1 r_{t-1} + \beta_2 r_{t-2} + \cdots + \beta_p r_{t-p} - \beta_1 r_{t-2} - \beta_2 r_{t-3} - \ldots - \beta_p r_{t-p-1} \quad (26)$$

Summing Equation (23) and (26), we have

$$-B(1)r_{t-1} + \sum_{i=1}^{p} \beta_i\, \Delta r_{t-i} = -r_{t-1} + \beta_1 r_{t-2} + \beta_2 r_{t-3} + \ldots + \beta_p r_{t-p-1} + \beta_1 r_{t-1} + \beta_2 r_{t-2} + \cdots + \beta_p r_{t-p} - \beta_1 r_{t-2} - \beta_2 r_{t-3} - \cdots - \beta_p r_{t-p-1} \quad (27)$$

$$-B(1)r_{t-1} + \sum_{i=1}^{p} \beta_i\, \Delta r_{t-i} = -r_{t-1} + \beta_1 r_{t-1} + \beta_2 r_{t-2} + \ldots + \beta_p r_{t-p} \quad (28)$$

Renovating Equation (28) gives back Equation (18a).
Thus,

$$-B(1)r_{t-1} + \sum_{i=1}^{p} \beta_i\, \Delta r_{t-i} + r_{t-1} = \sum_{i=1}^{p} \beta_i\, r_{t-i}$$

To obtain Equation (18b):
$\sum_{i=0}^{n} a_i'\, g_{t-i} = B(1)\beta' g_t + \sum_{i=1}^{n} a_i{}'\Delta g_{t-i}$,
We consider,

$$\beta = \frac{a(L)}{B(L)} => B(L)\beta = a(L) \quad (28a)$$

$$B(L)\beta'\Delta g_t = a'(L)\Delta g_t \quad (28b)$$

$= a'(L)(g_t - g_{t-1})$

$= a'(L)(g_t - Lg_t)$ (28c)

$if\ we\ assume\ L = 1\ in\ Equation\ (28c)\ then,$

$B(1)\beta'\Delta g_t = a'(1)(g_t - g_t) = 0$ (28d)

But,

$B(L)\beta' g_{t-1} = a'(L)\, g_{t-1}$ (28e)

$B(L)\beta' g_{t-1} = \sum_{i=0}^{n} a_i'\, L^i\, g_{t-1}$ (28f)

$B(L)\beta' L g_t = \sum_{i=0}^{n} a_i'\, L^i\, g_{t-1}$ (28g)

$B(L)\beta' L g_t = \sum_{i=0}^{n} a_i'\ g_{t-i-1}$ (28h)

$if\ L = 1$ to the left hand side of Equation (28h), gives

$B(1)\beta' g_t = a'_0 g_{t-1} + a'_1 g_{t-2} + a'_2 g_{t-3} + \ ...\ + a'_n g_{t-n-1}$ (29)

Thus $\sum_{i=0}^{n} a_i'\, \Delta g_{t-i} = a'_0 \Delta g_t + a'_1 \Delta g_{t-1} + \ ...\ + a'_n \Delta g_{t-n}$ (30)

$= a'_0(g_t - g_{t-1}) + a'_1(g_{t-1} - g_{t-2}) + ... + a'_n(g_{t-n} - g_{t-n-1})$ (31)

Thus,

$\sum_{i=0}^{n} a_i'\, \Delta g_{t-i} = a'_0 g_t - a'_0 g_{t-1} + a'_1 g_{t-1} - a'_1 g_{t-2} + ... + a'_n g_{t-n} - a'_n g_{t-n-1}$ (32)

Summing Equation (29) and (32), gives Equation (18b);

$\sum_{i=0}^{n} a_i'\, g_{t-i} = B(1)\beta' g_t + \sum_{i=0}^{n} a_i'\, \Delta g_{t-i}$

$$\sum_{i=0}^{n} a_i'\, g_{t-i} = a'_0 g_{t-1} + a'_1 g_{t-2} + a'_2 g_{t-3} + \ ...\ + a'_n g_{t-n-1} + a'_0 g_t - a'_0 g_{t-1} + a'_1 g_{t-1} - a'_1 g_{t-2} + ... + a'_n g_{t-n} - a'_n g_{t-n-1}$$

$\sum_{i=0}^{n} a_i'\, g_{t-i} = a'_0 g_{t-0} + a'_1 g_{t-1} + + a'_2 g_{t-2} + \cdots + a'_n g_{t-n}$

Clearly, from Equation (15) substituting Equation (18a) and Equation (18b) in Equation (15) yield;

$r_t = \beta_0 - B(1) r_{t-1} + \sum_{i=1}^{p} \beta_i\, \Delta r_{t-i} + r_{t-1} + B(1)\beta' g_t + \sum_{i=0}^{n} a_i'\, \Delta g_{t-i} + \varepsilon_t$ (33)

$r_t - r_{t-1} = \beta_0 - B(1) r_{t-1} + \sum_{i=1}^{p} \beta_i\, \Delta r_{t-i} + B(1)\beta'(g_{t-1} + \Delta g_t) + \sum_{i=0}^{n} a_i'\, \Delta g_{t-i} + \varepsilon_t$ (34)

Where $\Delta g_t = g_t - g_{t-1} \Rightarrow g_t = g_{t-1} + \Delta g_t$ (35)

$r_t - r_{t-1} = \beta_0 - B(1) r_{t-1} + B(1)\beta' g_{t-1} + \sum_{i=1}^{p} \beta_i\, \Delta r_{t-i} + B(1)\beta' \Delta g_t + \sum_{i=0}^{n} a_i'\, \Delta g_{t-i} + \varepsilon_t$ (36)

$and\ B(1)\beta'\Delta g_t = a(1)(g_t - g_t) = 0$, from Equation (28d)

Where $\Delta r_t = r_t - r_{t-1}$ from first difference forward operator

Then,

$\Delta r_t = \beta_0 - B(1) r_{t-1} + B(1)\beta' g_{t-1} + \sum_{i=1}^{p} \beta_i\, \Delta r_{t-i} + a(1)(g_t - g_t) + \sum_{i=0}^{n} a_i'\, \Delta g_{t-i} + \varepsilon_t$ (37)

$\Delta r_t = \beta_0 - B(1) r_{t-1} + B(1)\beta' g_{t-1} + \sum_{i=1}^{p} \beta_i\, \Delta r_{t-i} + 0 + \sum_{i=0}^{n} a_i'\, \Delta g_{t-i} + \varepsilon_t$ (38)

$setting\ -B(1) = \Upsilon_1\ and\ B(1)\beta' = \Upsilon_2,\ so\ that\ Equation\ (38)\ becomes$

$\Delta r_t = \beta_0 + \sum_{i=1}^{p} \beta_i\, \Delta r_{t-i} + \sum_{i=0}^{n} a_i'\, \Delta g_{t-i} + \Upsilon_1 r_{t-1} + \Upsilon_2 g_{t-1} + \varepsilon_t$ (39)

Equation (39) is the ARDL (p, n) unrestricted model (UM) that examines the long-run and short-run relationships between the variables.
Equation (39) can be extended to ARDL (p, n, m, o, q, v) as follows:

$$\Delta r_t = \beta_0 + \sum_{i=1}^{p} \beta_i \Delta r_{t-i} + \sum_{i=0}^{n} a_i' \Delta g_{t-i} + \sum_{i=0}^{m} \theta_i' \Delta t_{t-i} + \sum_{i=0}^{o} \alpha_i' \Delta m_{t-i} + \sum_{i=0}^{q} \lambda_i' \Delta e_{t-i} + \sum_{i=0}^{v} \mu_i' \Delta s_{t-i} + \Upsilon_1 r_{t-1} + \Upsilon_2 g_{t-1} + \Upsilon_3 t_{t-1} + \Upsilon_4 m_{t-1} + \Upsilon_5 e_{t-1} + \Upsilon_6 s_{t-1} + \varepsilon_t \quad (40)$$

Where $-B(1) = \Upsilon_1$, $B(1)\beta' = \Upsilon_2$, $B(1)\rho' = \Upsilon_3$, $B(1)\delta' = \Upsilon_4, B(1)\varphi' = \Upsilon_5, B(1)\omega' = \Upsilon_6$, equation (40) is a consequence of Equation (18a) to Equation (18f).
$\beta_0 is\ the\ constant\ term,\ where\ \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5\ and\ \Upsilon_6$ are the Long- run parameters. $\beta, a, \theta, \alpha, \lambda, and\ \mu$ are the short- run parameters of the ARDL UM respectively.

Therefore, the general unrestricted error correction model (UECM) of Equation (40) is as follows:

$$\Delta r_t = \beta_0 + \sum_{i=1}^{p} \beta_i \Delta r_{t-i} + \sum_{i=0}^{n} a_i' \Delta g_{t-i} + \sum_{i=0}^{m} \theta_i' \Delta t_{t-i} + \sum_{i=0}^{o} \alpha_i' \Delta m_{t-i} + \sum_{i=0}^{q} \lambda_i' \Delta e_{t-i} + \sum_{i=0}^{v} \mu_i' \Delta s_{t-i} + \Upsilon Z_{t-1} + \xi_t \quad (41)$$

Υ is the error correction parameter, Z_{t-1} is the residuals that are obtained from the estimated cointegration model and ξ_t is the disturbance term assumed to be uncorrelated with zero means.

Equation (41) is the error correction model of ARDL that gives adjustment back to the long – run stability. Υ is the error correction parameter, Z_{t-1} is the residuals that are obtained from the estimated cointegration model. The lagged error correction term Z_{t-1} derived from the Error Correction Model is an important element in the dynamics of cointegration system as it allows for adjustment back to the long - run equilibrium relationship given a deviation in the last quarter (Irefin and Yaaba (2013), Shahbaz and Faridul (2011)). and ξ_t is the disturbance term assumed to be uncorrelated with zero means.
From Equation (17), if we let $\varepsilon_t = z_t$, we have:

$$r_t = \alpha_0 + \alpha_1 g_t + \alpha_2 t_t + \alpha_3 m_t + \alpha_4 e_t + \alpha_5 s_t + z_t \quad (42)$$

From Equation (4212), Z_t is the error correction term in the Ordinary Least-Squares residuals series from the Long-run cointegration regression.

If we lag Equation (42) by one, that is L$r_t = r_{t-1}$ and L is the lag operator, then we have

$$r_{t-1} = \alpha_0 + \alpha_1 g_{t-1} + \alpha_2 t_{t-1} + \alpha_3 m_{t-1} + \alpha_4 e_{t-1} + \alpha_5 s_{t-1} + z_{t-1} \quad (43)$$

$$z_{t-1} = r_{t-1} - \alpha_0 - \alpha_1 g_{t-1} - \alpha_2 t_{t-1} - \alpha_3 m_{t-1} - \alpha_4 e_{t-1} - \alpha_5 s_{t-1} \quad (44)$$

Thus, substituting Equation (44) into Equation (18) we have,

$$\Delta r_t = \beta_0 + \sum_{i=1}^{p} \beta_i \Delta r_{t-i} + \sum_{i=0}^{n} a_i' \Delta g_{t-i} + \sum_{i=0}^{m} \theta_i' \Delta t_{t-i} + \sum_{i=0}^{o} \alpha_i' \Delta m_{t-i} + \sum_{i=0}^{q} \lambda_i' \Delta e_{t-i} + \sum_{i=0}^{v} \mu_i' \Delta s_{t-i} + \Upsilon(r_{t-1} - \alpha_0 - \alpha_1 g_{t-1} - \alpha_2 t_{t-1} - \alpha_3 m_{t-1} - \alpha_4 e_{t-1} - \alpha_5 s_{t-1}) + \xi_t \quad (45)$$

$$\Delta r_t = \beta_0 + \sum_{i=1}^{p} \beta_i \Delta r_{t-i} + \sum_{i=0}^{n} a_i' \Delta g_{t-i} + \sum_{i=0}^{m} \theta_i' \Delta t_{t-i} + \sum_{i=0}^{o} \alpha_i' \Delta m_{t-i} + \sum_{i=0}^{q} \lambda_i' \Delta e_{t-i} + \sum_{i=0}^{v} \mu_i' \Delta s_{t-i} + \Upsilon r_{t-1} - \Upsilon \alpha_0 - \Upsilon \alpha_1 g_{t-1} - \Upsilon \alpha_2 t_{t-1} - \Upsilon \alpha_3 m_{t-1} - \Upsilon \alpha_4 e_{t-1} - \Upsilon \alpha_5 s_{t-1} + \xi_t \quad (46)$$

From assumption (iii) we have,

$$\beta_0 - \Upsilon\alpha_0 = \rho_0, \Upsilon = \vartheta_1, -\Upsilon\alpha_1 = \vartheta_2, -\Upsilon\alpha_2 = \vartheta_3, -\Upsilon\alpha_3 = \vartheta_4, -\Upsilon\alpha_4 = \vartheta_5\ and - \Upsilon\alpha_5 = \vartheta_6.$$

Thus,

$$\Delta r_t = \rho_0 + \sum_{i=1}^{p} \beta_i \Delta r_{t-i} + \sum_{i=0}^{n} a_i' \Delta g_{t-i} + \sum_{i=0}^{m} \theta_i' \Delta t_{t-i} + \sum_{i=0}^{o} \alpha_i' \Delta m_{t-i} + \sum_{i=0}^{q} \lambda_i' \Delta e_{t-i} + \sum_{i=0}^{v} \mu_i' \Delta s_{t-i} + \vartheta_1 r_{t-1} + \vartheta_2 g_{t-1} + \vartheta_3 t_{t-1} + \vartheta_4 m_{t-1} + \vartheta_5 e_{t-1} + \vartheta_6 s_{t-1} + \xi_t \quad (47)$$

Equation (47) now accommodates both the long – run parameters ($\vartheta_1, \vartheta_2, \ldots, \vartheta_6$) and the short – run parameters ($\beta_i, a_i', \theta_i', \alpha_i', \lambda_i', \mu_i'$) respectively. The maximum lag length of Equation (47) can be selected using information criteria. Therefore, several information criteria have shown great reliability in the selection of a statistical model. In time series analysis there are different information criteria for their respective model and all criteria are likelihood based with two major parts or components. The first part tackles the goodness of fit of the model to the data, whereas the other component penalizes more heavily complicated models (Tsay 2014). The goodness of fit of a model is often measured by the maximum likelihood. The selection of the penalty is relatively subjective, that is different penalties for different information criterion.

The Akaike information criterion (1974 and 1976), Schwarz information criterion (1978) and the Hanan-Quinn (1979) information criterion for selecting the most parsimonious model. Over-parameterization is taking care of by this process (Campos *et al*. (2005)) as cited in (Irefin and Yaaba, 2013). Raykov and Marcoulides (1999) stated that by the principle of parsimony, it is expedient to estimate the model that includes minimum lag the present absence of residual autocorrelation. The information criteria are;

Akaike (AIC): $c_T(k) = -\frac{2ln(L_T(k))}{T} + 2\frac{k}{T}$,

Schwarz (SIC): $c_T(k) = -\frac{2ln(L_T(k))}{T} + k\frac{l_n(T)}{T}$,

Hanan-Quinn (HQ): $c_T(k) = -\frac{2ln(L_T(k))}{T} + 2k\frac{l_n(l_n(T))}{T}$

respectively. These criteria take the general form

$$c_T(k) = -\frac{2ln(L_T(k))}{T} + k\frac{\Omega(T)}{T},$$

Where $\Omega(T) = 2$ in the Akaike case, $\Omega(T) = l_n(T)$ in the Schwarz case and $\Omega(T) = 2l_n(l_n(T))$ in the Hanan-Quinn case, the likelihood function $L_T(k) = -\frac{T}{2}(1 + \log(2\pi) + \log\left(\frac{\hat{e}'\hat{e}}{T}\right))$ and $\hat{e}'\hat{e} = e^2 = \sum_{t=1}^{T}(lR - X_i'b)$, where X_i' is a vector of exogenous variables and b is a vector of coefficients such that;

$$X_i' = (x_1\ x_2\ x_3 \ldots x_k), \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_k \end{pmatrix}$$

From assumption (iv) above, Equation (47) becomes;

$$\Delta r_t = \rho_0 + \sum_{i=1}^{2} \beta_i \Delta r_{t-i} + \sum_{i=0}^{3} a_i' \Delta g_{t-i} + \sum_{i=0}^{0} \theta_i' \Delta t_{t-i} + \sum_{i=0}^{0} \alpha_i' \Delta m_{t-i} + \sum_{i=0}^{0} \lambda_i' \Delta e_{t-i} + \sum_{i=0}^{1} \mu_i' \Delta s_{t-i} + \vartheta_1 r_{t-1} + \vartheta_2 g_{t-1} + \vartheta_3 t_{t-1} + \vartheta_4 m_{t-1} + \vartheta_5 e_{t-1} + \vartheta_6 s_{t-1} + \xi_t \quad (48)$$

$$\Delta r_t = \rho_0 + \beta_1 \Delta r_{t-1} + \beta_2 \Delta r_{t-2} + a_0' \Delta g_t + a_1' \Delta g_{t-1} + a_2' \Delta g_{t-2} + a_3' \Delta g_{t-3} + \theta_0' \Delta t_t + \alpha_0' \Delta m_t + \lambda_0' \Delta e_t + \mu_0' \Delta s_t + \mu_1' \Delta s_{t-1} + \vartheta_1 r_{t-1} + \vartheta_2 g_{t-1} + \vartheta_3 t_{t-1} + \vartheta_4 m_{t-1} + \vartheta_5 e_{t-1} + \vartheta_6 s_{t-1} + \xi_t \quad (49)$$

where $\Delta r_t = r_t - r_{t-1}$
$r_t = \rho_0 + r_{t-1} + \beta_1(r_{t-1} - r_{t-2}) + \beta_2(r_{t-2} - r_{t-3}) + a_0'(g_t - g_{t-1}) + a_1'(g_{t-1} - g_{t-2}) + a_2'(g_{t-2} - g_{t-3}) + a_3'(g_{t-3} - g_{t-4}) + \theta_0'(t_t - t_{t-1}) + \alpha_0'(m_t - m_{t-1}) + \lambda_0'(e_t - e_{t-1}) + \mu_0'(s_t - s_{t-1}) + \mu_1'(s_{t-1} - s_{t-2}) + \vartheta_1 r_{t-1} + \vartheta_2 g_{t-1} + \vartheta_3 t_{t-1} + \vartheta_4 m_{t-1} + \vartheta_5 e_{t-1} + \vartheta_6 s_{t-1} + \xi_t$ (50)

Also from assumption (v) Equation (50) becomes;
$r_t = \rho_0 + (1 + \beta_1 + \vartheta_1)r_{t-1} + (\beta_2 - \beta_1)r_{t-2} + (a_1' - a_0' + \vartheta_2)g_{t-1} + (a_3' - a_2')g_{t-3} + \theta_0' t_t + \alpha_0' m_t + \lambda_0' e_t + (\vartheta_6 + \mu_1' - \mu_0')s_{t-1} + \xi_t$ (51)
Where $\rho_0 = C, (1 + \beta_1 + \vartheta_1) = C(1), (\beta_2 - \beta_1) = C(2), (a_1' - a_0' + \vartheta_2) = C(3), (a_3' - a_2') = C(4), \theta_0' = C(5), \alpha_0' = C(6), \lambda_0' = C(7), (\vartheta_6 + \mu_1' - \mu_0') = C(8)$
$LogR_t = C + C(1)LogR_{t-1} + C(2)LogR_{t-2} + C(3)LogG_{t-1} + C(4)LogG_{t-3} + C(5)LogT_t + C(6)LogM_t + C(7)LogE_t + C(8)LogS_{t-1} + \xi_t$ (52)

Equation (52) is the modified ARDL model to be estimated using least squares (LS). Then,
$C = 1, C(1) = 2, C(2) = 3, C(3) = 4, C(4) = 5, C(5) = 6, C(6) = 7, C(7) = 8 \; and \; C(8) = 9$

Thus,

$$K = \{1, 2, 3, 4,5,6,7,8,9\} \;\; n(K) = 9$$

The elements of the set K are the parameters of the long – run stability for Equation (52) and the number of elements in the set K represents the order of Equation (52). Therefore, $n(K)$ now paved way in applying the binomial coefficient model to determine the total number of model each order can accommodate and their respective stability for different families of r.

The Binomial Coefficients Model

The binomial theorem is given by;

$$(a + b)^i = \sum_{r=0}^{i} \binom{i}{r} a^{i-r} b^r$$

From the above theorem, assumption (vi) $a = b = 1, i = n(K), n(K)$ is the number of elements in set K and it takes positive numerical values. Thus,

$$(1 + 1)^{n(K)} = \sum_{r=0}^{n(K)} \binom{n(K)}{r}$$

Then,

$$2^{n(K)} = C_0^{n(K)} + \sum_{r=1}^{n(K)} \binom{n(K)}{r}$$

$$2^{n(K)} - C_0^{n(K)} = \sum_{r=1}^{n(k)} \binom{n(K)}{r}$$

Where $C_0^{n(K)} = 1$, implies that

$$2^{n(K)} - 1 = \sum_{r=1}^{n(K)} \binom{n(K)}{r}$$

T_{nrm} (Total number of regression model) $= 2^{n(K)} - 1$

$$= \sum_{r=1}^{n(K)} \binom{n(K)}{r}$$

T_{nrm} (Total number of regression model) $= \sum_{r=1}^{n(K)} \binom{n(K)}{r}$, $r = 1(1)n(K)$ (53)

Using one of the properties of combinatorial coefficients;

$$\binom{n}{j} + \binom{n}{j+1} = \binom{n+1}{j+1} \qquad (*)$$

From equation (*)

$$\binom{n}{j} = \binom{n+1}{j+1} - \binom{n}{j+1} \qquad (**)$$

Using assumptions (viii) and (ix) in equation **

$$\binom{n(k)-1}{r-1} = \binom{n(k)}{r} - \binom{n(k)-1}{r} \quad (***)$$

Equation (***) gives the number of models with fixed intercept. From equation (**) decompose $\binom{n}{j}$ such that,

$$\binom{n}{j} = \binom{n-1}{j} - \binom{n-1}{j-1} \quad (****)$$

Substitute equation (****) into equation (**)
Thus,

$$\binom{n-1}{j-1} = \binom{n+1}{j+1} - \binom{n}{j+1} - \binom{n-1}{j} \quad (*****)$$

Using assumptions (viii) and (xi) to obtain the stability rows for different order of $n(k)$ and their respective families of r.

$$\binom{n(k)-2}{r-2} = \binom{n(k)}{r} - \binom{n(k)-1}{r} - \binom{n(k)-2}{r-1} \quad (******)$$

Equation (53) gives the total number of regression model for different order of $n(K)$ for the long – run parameters. For a fixed $C = 1$ in Equation (53), it reduces T_{nrm} for which the stability of the models is obtained (see Appendix 3). In the estimation of Equation (52), if some of its parameters are statistically insignificant in their p – values, then we introduce the matrix elimination model.

Matrix Elimination Model

The modified ARDL model of Equation (52) is the least parsimonious model as suggested by the information criteria, which must have long run stability but with some parameters that are statistically insignificant at a specified level of percentage in their p-values (see Appendix 2). Thereafter, the matrix elimination model starts by eliminating at a time the least significant variable form the model whose p-value is greater than a specified level of significance. When this is done, the reduced ARDL model must also be stable. The process of matrix elimination is repeated for several times until no p-value in their respective parameters are greater than a specified significant level. Then the process of the matrix elimination is terminated for which all the parameters in their p-value are less than a specified level of significance. The model for which all the p-values are less than a specified significance level, perhaps at 5% is statistically significant for parameter estimation. In the other hand, the process of matrix elimination is terminated when the model for which the variable and its parameter is eliminated becomes unstable, and then the preceding model is adopted as the choice model irrespective of whether there are parameters that are statistically insignificant.

From Equation (52)

$$LogR_t = C(1)LogR_{t-1} + C(2)LogR_{t-2} + C(3)LogG_{t-1} + C(4)LogG_{t-3} + C(5)LogT_t + C(6)LogM_t + C(7)LogE_t + C(8)LogS_{t-1} + C$$

is transformed to a matrix $M_{n\times m}$ and ɣ (also see Appendix 2) such that;

$$LogR_t = (LogR_{t-1}\ LogR_{t-2}\ LogG_{t-1}\ LogG_{t-3}\ LogT_t\ LogM_t\ LogE_t\ LogS_{t-1}\ 1)(C(1)\ C(2)\ C(3)\ C(4)\ C(5)\ \ldots\ C)^T \quad (54)$$

$$LogR_t = M_{n\times m}ɣ \quad (55)$$

Thus,

$$M_{m\times n}^T LR_{n\times 1} = M_{m\times n}^T M_{n\times m}ɣ \quad (56)$$

for which $C(1) = ɣ_1, C(2) = ɣ_2, C(3) = ɣ_3, \ldots, C = ɣ_m$ and $LogR_t = LR_{n\times 1}$

Therefore, $M_{(n\times m)}$ must be stable in all its parameters, $ɣ_j, j = 1(1)m$

Where ɣ is the unknown parameter, j=1(1) m, i=1(1) n.

Then the product,

$M_{m\times n}^T M_{n\times m} = Z_{m\times m}$ and $M_{m\times n}^T LR_{n\times 1} = H_{m\times 1}$,

$$Z_{(m\times m)}ɣ = H_{m\times 1}$$

Then,

$$ɣ = Z_{m\times m}^{-1} H_{m\times 1}$$

$$ɣ = ɣ_{m\times 1} \quad (57)$$

$$ɣ = \begin{bmatrix} ɣ_1 \\ ɣ_2 \\ ɣ_3 \\ ɣ_4 \\ ɣ_5 \\ ɣ_6 \\ . \\ . \\ . \\ ɣ_m \end{bmatrix}$$

If there exist any $ɣ_j$, *say* $ɣ_{m-1}$ in ɣ whose p - value > 0.05 , then there is need to carry out matrix elimination on the exogenous variable and its parameter value at the same time from the matrix $M_{(n\times m)}$ and its transpose. The reason for this matrix elimination is to obtain parameters that are stable and statistically significant at 5%, hence the result gives estimates that are statistically significant for parameter estimation. Thus, the new system becomes;

$$\dot{M}_{(m-1)\times n}^T, \dot{M}_{n\times(m-1)}, LR_{(n\times 1)}\ and\ \dot{ɣ}$$

Therefore, $\dot{M}_{n\times(m-1)}$ must be stable in all its parameters, $ɣ_j\ j = 1(1)m - 1$.

If m_{ij} is not statistically significant at 5% in $M_{(n\times m)}$, then the variable should be eliminated from the matrix $M_{(n\times m)}$ and $M^T_{m\times n}$. Where m_{ji}, m_{ij} had been removed from the matrix $M_{(n\times m)}$ and $M^T_{m\times n}$ respectively. Similarly, γ_{m-3} from γ , then the first elimination process becomes;

$$\dot{M}^{\mathrm{T}}_{(\mathrm{m}-1)\times\mathrm{n}}\dot{M}_{n\times(m-1)}\dot{\gamma} \quad = \dot{M}^{T}_{(m-1)\times n}LR_{(n\times1)}$$

Where,

$$\dot{M}^{\mathrm{T}}_{(m-1)\times\mathrm{n}}\dot{M}_{n\times(m-1)} = \dot{Z}_{(m-1)\times(m-1)}$$

and

$$\dot{M}^{T}_{(m-1)\times n}LR_{(n\times1)} = \dot{H}_{(m-1)\times1}$$

Thus,

$$\dot{Z}_{(m-1)\times(m-1)}\dot{\gamma} \quad = \dot{H}_{(m-1)\times1}$$

Then to obtain $\dot{\gamma}$, the matrix becomes

$$\dot{\gamma} = \dot{Z}_{(m-1)\times(m-1)}{}^{-1}\dot{H}_{(m-1)\times1}$$

$$\dot{\gamma} = \gamma_{(m-1)\times1} \qquad (58)$$

If there exist any $\dot{\gamma}_j$, say $\dot{\gamma}_{m-2}$ whose p - value > 0.05 in all its parameter values $\dot{\gamma}_j\ j = 1(1)m-1$, then there is need to carry out matrix elimination on the exogenous variable and its parameter value at the same time from the matrix $\dot{M}_{n\times(m-1)}$ and its transpose. The reason for this matrix elimination is to obtain parameters that are stable and statistically significant at 5%, hence the result gives estimates that are statistically significant for parameter estimation. Thus, the new system becomes;

$$\ddot{M}^{T}_{(m-2)\times n}\,,\ddot{M}_{n\times(m-2)}\,,LR_{(n\times1)}\ and\ \ddot{\gamma}$$

Therefore, $\ddot{M}_{n\times(m-2)}$ must be stable in all its parameters, where s_{ji}, s_{ij} had been removed from the matrix $\dot{M}_{n\times(m-1)}$ and $\dot{M}^{\mathrm{T}}_{(m-1)\times\mathrm{n}}$ respectively. Similarly, $\dot{\gamma}_{m-2}$ from $\dot{\gamma}$.

Where the second elimination process becomes;

$$\ddot{M}^{T}_{(m-2)\times n}\ddot{M}_{n\times(m-2)}\ddot{\gamma} \quad = \ddot{M}^{T}_{(m-2)\times n}\,LR_{(n\times1)}$$

Where

and

$$\ddot{M}^{T}_{(m-2)\times n}LR_{(n\times1)} = \ddot{H}_{(m-2)\times1}$$

Thus,

$$\ddot{Z}_{(m-2)\times(m-2)}\ddot{\gamma} \quad = \ddot{H}_{(m-2)\times1}$$

$$\ddot{\gamma} = \ddot{Z}_{(m-2)\times(m-2)}{}^{-1}\ddot{H}_{(m-2)\times 1}$$

$$\ddot{\gamma} = \gamma_{(m-2)\times 1} \qquad (59)$$

Therefore, $\ddot{M}_{n\times(m-2)}$ must be stable in all its parameters, $\ddot{\gamma}_j$, $j = 1(1)m-2$.
If there exist any $\ddot{\gamma}_j$, $say\ \ddot{\gamma}_{m-3}$ whose p – value > 0.05 , then there is need to carry out matrix elimination on the exogenous variable and its parameter value at the same time from the matrix $\ddot{M}_{n\times(m-2)}$ and its transpose. The reason for this matrix elimination is to obtain parameters that are stable and statistically significant at 5%, hence the result gives estimates that are statistically significant for parameter estimation. Thus, the new system becomes;

$$\dddot{M}^T_{(m-3)\times n}, \dddot{M}_{n\times(m-3)}, LR_{(n\times 1)}\ and\ \dddot{\gamma}$$

$$\dddot{M}^T_{(m-3)\times n}\ \dddot{M}_{n\times(m-3)}\dddot{\gamma} = \dddot{M}^T_{(m-3)\times n}\ LR_{(n\times 1)}$$

Where,

$$\dddot{M}^T_{(m-3)\times n}\ \dddot{M}_{n\times(m-3)} = \dddot{Z}_{(m-3)\times(m-3)}$$

Similarly,

$$\dddot{M}^T_{(m-3)\times n}\ LR_{(n\times 1)} = \dddot{H}_{(m-3)\times 1}$$

Thus,

$$\dddot{Z}_{(m-3)\times(m-3)}\dddot{\gamma} = \dddot{H}_{(m-3)\times 1}$$

$$\dddot{\gamma} = \dddot{\gamma}_{(m-3)\times 1} \qquad (60)$$

$$\dddot{\gamma}^T = [\dddot{\gamma}_1 \quad \dddot{\gamma}_2 \quad \dddot{\gamma}_3 \quad \dddot{\gamma}_4 \quad \dddot{\gamma}_5 \quad \cdot \quad \cdot \quad \cdot \quad \dddot{\gamma}_{m-3}]$$

DATA/ANALYSIS OF THE METHOD

The Modified ARDL Model

We analyzed the modified ARDL model of Equation (52) with the aid of econometric view (EView) package applying least squares to estimate the long – run parameters of order nine using quarterly data from CBN annual statistical bulletin that span from 1999Q1 to 2011Q2 and 2000Q1 to 2012Q4 (see Table 1 in Appendix 1). It was observed that the exogenous variables such as second lag of reserve, monetary policy rate, exchange rate are inverse related to reserves and others are positive related. The estimated parameters shows that the major determinant to foreign reserves are first and second lag of GDP and Trade Openness. In the estimated parameters for period (2000Q1- 2012Q4) only monetary policy rate is statistically insignificant and for period (1999Q1 – 2011Q2) monetary policy rate, exchange rate and foreign debt are statistically insignificant since they have p – values greater than 5% level of significance (see Table 1and 3 in Appendix 3). The adjustment back to the long – run stability is about 19.1% and 23% disequilibrium is corrected on a quarterly basis with variation in reserves (also see Table 2 and 4 in Appendix 3). The estimation of parameters in both periods shows long – run stability in their cumulative sum of recursive residuals (CUSUM) and cumulative sum of squares of recursive residuals (CUSUMSQ) at 5% respectively (figures 1, 2, 3 and 4 in Appendix 3).

The Binomial Coefficients Model

In ARDL model of any order say $n(K)$, the long – run stability follows a sequence of pattern of the binomial coefficients model. That is the stability for order $n(K)$ follows a pattern around $n(K-2)$of the binomial coefficients model. Similarly, the stability for order $n(K-1)$ follows a pattern around $n(K-3)$ of the binomial coefficient model and the stability for order $n(K-2)$ follows a pattern around $n(K-4)$ of the binomial coefficients model and so on up to the stability for order $n(K-6)$ which also follows a pattern around $n(K-8)$. We observed that there is a synergy between the stability obtained from the binomial coefficients model and the econometric view package (see Table 5 and 6 in Appendix 3).

The Matrix Elimination Model for Sample (2000Q1-2012Q4)

The modified ARDL model of Equation (57) for order $n(K) = 9$ has its estimated parameters and respective p – values as;

$$\gamma = \begin{bmatrix} 1.180918 \\ -0.344818 \\ 0.658327 \\ 0.657772 \\ 0.053448 \\ -0.144597 \\ -0.781727 \\ 0.114542 \\ -11.03724 \end{bmatrix} \quad \text{With respective} \quad p - values = \begin{bmatrix} 0.0000 \\ 0.0142 \\ 0.0001 \\ 0.0002 \\ 0.0004 \\ 0.0654 \\ 0.0038 \\ 0.0140 \\ 0.0000 \end{bmatrix}$$

LR = 1.18091777719 LR (-1) - 0.344817570867 LR (-2) + 0.658326678092 LG (-1) + 0.657771795343 LG (-3) + 0.0534476224896 LT - 0.144596977585 LM - 0.781727112739 LE + 0.11454201159 LS (-1) -11.0372378768

For other subsequent elimination process for equation (58), (59) and (60) gives;

$$\dot{\gamma} = \begin{bmatrix} 1.238195 \\ -0.380701 \\ 0.553822 \\ 0.551459 \\ 0.046226 \\ -0.597976 \\ 0.064158 \\ -9.504126 \end{bmatrix} \quad \text{With respective} \quad p - values = \begin{bmatrix} 0.0000 \\ 0.0081 \\ 0.0006 \\ 0.0008 \\ 0.0016 \\ 0.0179 \\ 0.0897 \\ 0.0002 \end{bmatrix}$$

LR = 1.23819509391 LR (-1) - 0.38070101803 LR (-2) + 0.5538219435 LG (-1) + 0.55145886501 LG (-3) + 0.0462256788374 LT - 0.59797561966 LE + 0.0641575429724 LS (-1) - 9.50412628637

$$\ddot{\gamma} = \begin{bmatrix} 1.331784 \\ -0.515163 \\ 0.463629 \\ 0.458765 \\ 0.046608 \\ -0.381848 \\ -7.385031 \end{bmatrix} \quad \text{With respective} \quad p - values = \begin{bmatrix} 0.0000 \\ 0.0001 \\ 0.0023 \\ 0.0030 \\ 0.0018 \\ 0.0804 \\ 0.0008 \end{bmatrix}$$

LR = 1.33178401462 LR (-1) - 0.515162841344 LR (-2) + 0.463628536411 LG (-1) + 0.458765210769 LG (-3) + 0.0466082670768LT - 0.381848461658LE -7.38503058454

$$\dddot{\gamma} = \begin{bmatrix} 1.395724 \\ -0.533517 \\ 0.300764 \\ 0.291693 \\ 0.040314 \\ -5.764844 \end{bmatrix}, \quad \text{With respective} \quad p - values = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0109 \\ 0.0148 \\ 0.0058 \\ 0.0039 \end{bmatrix}.$$

LR = 1.39572363349 LR (-1) - 0.533517479032 LR (-2) + 0.300764344044 LG (-1) + 0.291692664665 LG (-3) + 0.0403138779589 LT - 5.76484381234

Matrix Elimination Model for Sample (1999Q1-2011Q2)

Parameters for the full matrix and the respective p-values of Equation (57)

$$\gamma = \begin{bmatrix} 1.337251 \\ -0.502723 \\ 0.519842 \\ 0.523464 \\ 0.056753 \\ -0.153850 \\ -0.366501 \\ 0.077155 \\ -9.456340 \end{bmatrix} \quad \text{With respective} \; p - values = \begin{bmatrix} 0.0000 \\ 0.0006 \\ 0.0070 \\ 0.0104 \\ 0.0012 \\ 0.1568 \\ 0.1248 \\ 0.1229 \\ 0.0038 \end{bmatrix}$$

LR = 1.33725141305 LR (-1) - 0.502723321647 LR (-2) + 0.51984245871 LG (-1) + 0.523464485408 LG (-3) + 0.0567532948224 LT - 0.153849882343 LM - 0.366501008793 LE + 0.0771553543495 LS (-1) - 9.45633965002

For other subsequent elimination process for Equations (58), (59) and (60) gives;

$$\dot{\gamma} = \begin{bmatrix} 1.395996 \\ -0.559112 \\ 0.502722 \\ 0.505226 \\ 0.048285 \\ -0.282676 \\ 0.035195 \\ -9.420892 \end{bmatrix} \quad \text{with respective} \quad p - values = \begin{bmatrix} 0.0000 \\ 0.0001 \\ 0.0096 \\ 0.0140 \\ 0.0030 \\ 0.2247 \\ 0.3829 \\ 0.0043 \end{bmatrix}$$

LR = 1.39599636451 LR (-1) - 0.559112362247 LR (-2) + 0.502722010068 LG (-1) + 0.505226486255 LG (-3) + 0.048285044094 LT - 0.282676270411 LE + 0.0351953696723 LS (-1) - 9.420892416

$$\ddot{\mathrm{y}} = \begin{bmatrix} 1.428055 \\ -0.607328 \\ 0.439320 \\ 0.435513 \\ 0.047953 \\ -0.175127 \\ -7.876563 \end{bmatrix} \text{ with respective } p-values = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0133 \\ 0.0199 \\ 0.0031 \\ 0.3712 \\ 0.0038 \end{bmatrix}$$

LR = 1.42805536806 LR (-1) - 0.607327982857 LR (-2) + 0.439320321973 LG (-1) + 0.435513441743 LG (-3) + 0.047953118948 LT - 0.175127088498 LE - 7.87656294372

$$\dddot{\mathrm{y}} = \begin{bmatrix} 1.425515 \\ -0.575695 \\ 0.337152 \\ 0.327480 \\ 0.044101 \\ -6.512297 \end{bmatrix} \text{, with respective } p-values = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0109 \\ 0.0189 \\ 0.0043 \\ 0.0031 \end{bmatrix}$$

LR = 1.42551453898LR (-1) - 0.575695249874 LR (-2) + 0.337151836674 LG (-1) + 0.327479811785 LG (-3) + 0.0441008704776 LT - 6.51229686678

In the above matrix elimination model, all parameters from $\dddot{\mathrm{y}}$ are statistically significant at their respective p – values. Hence, can be used for statistical estimation.

INTERPRETATION OF RESULTS

The Binomial Coefficients Model and ARDL Model

The stability of an autoregressive distributed lag model with intercept and no constant trend twirl along or twisted on the sequence of pattern of the binomial coefficients model for different order of n(k). The whole process gives an insight of how each family present their respective parsimonious model taking **r** family at a time. Then, a selection for the general parsimonious model is carried out with a clearer description form econometric view package. The binomial coefficients model cannot really tell the least parsimonious model and as such we introduce econometric view which explains the process by exploring information criteria (AIC, SIC and HQ). Over-parameterization is taking care of by this process Campos *et al*. (2005). The findings show that the full matrix has few of its parameters that are statistically insignificant at 5%. The result estimating Foreign Reserves from the estimated parameters for data 2000Q1-2012Q4 and 1999Q1-2011Q2 are stable and a positive indication that the model is reliable for statistical estimations and it performs better to the estimated parameters obtained by Irefin and Yaaba (2013). Υ is the error correction parameter, Z_{t-1} is the residual that is obtained from the estimated cointegration model and ξ_t is the disturbance term assumed to be uncorrelated with zero means.

The results for short - run of Reserves on exogenous variables within the error correction model are examined by using Equation (41). The short-run alteration process is examined from the error correction model. If the coefficient of Z_{t-1} lies between 0 and -1, the correction to Reserves in the period t is a fraction of the error in period $t-1$.As a result, the Z_{t-1} causes Reserves to converge monotonically to its long-run equilibrium path in reply to changes in the exogenous variables. If the coefficient Z_{t-1} is positive or less than -2, this will cause the Reserves to diverge. If the value of the coefficient is between -1 and -2, Error Correction Model (ECM) will produce damped oscillations in the Reserves around its equilibrium path.

The Matrix Elimination Model

In the full matrix from our result, it was clear that few parameters were insignificant at 5% level of significance and the result obtained by Irefin and Yaaba (2013). Hence the need for matrix elimination on our model that is motivated by the p – values of the respective parameters. The systematic elimination now sprang a model whose p – values are statistically significant at 5%. Therefore, we say freely that the model now has its true determinants for statistical estimation.

CONCLUSION AND POLICY IMPLICATIONS

The result indicated by Irefin and Yaaba (2013) for R – squared explained over 98% movement in reserve and whereas ours explained over 99% for reserves (see Table 1 and 3 in Appendix 3). It was observed from our findings that gross domestic product (GDP) with first and third lags respectively exhibit a positive relationship with foreign reserves coinciding with the results from (Irefin and Yaaba (2013), Bankole and Shuaibu (2013), Mosayeb *et al* (2005)) and as such considered as the major determinant of foreign reserves. The distance second is trade openness with positive relationship and significant as reported by Atif *et al.* (2010). Again, it was found that monetary policy rates (MPR), Exchange rate (EXR) are inverse related to reserve, which again confirms Irefin and Yaaba (2013). By this, Ben – Bassat and Gottlied (1992) believes MPR could have been positive related and it shows from our findings that any percentage increase in MPR and EXR can drain foreign reserve in Nigeria (Table 1 and 3 in Appendix 3). The error correction parameters (-0.19126 and -0.22712) for the short – run dynamics are indicators towards long – run stability. Comparing the result of Irefin and Yaaba (2013) and our findings, it was obvious that about 19.1% and 23% disequilibrium is corrected as against 14% on a quarterly basis with variation in reserves. The implication of this is that, it will take disequilibrium to be corrected to equilibrium in five quarters and one month and four quarters, one month and two weeks respectively as against seven quarters and one month as obtained in Irefin and Yaaba (2013). Therefore, convergence towards equilibrium path in our findings is faster when compared with the result from Irefin and Yaaba (2013).

RECOMMENDATIONS

We therefore recommend that decision and policy makers formulate a mechanism to maintain long – run stability in Foreign Reserves for Nigeria. In this study intercept was fixed to the binomial coefficients model and further studies should embrace both intercept and constant trend fixed to the binomial coefficients model and investigation should be carried out to determine the difference in stability between the binomial coefficient model and the stability obtained from autoregressive distributed lag model.

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Appendix 1

TABLE 1: Data from CBN Annual Bulletin (2013)

Quarter	R	G	IM	M	T	E	S
1999Q1	5507.1	98099.48	167230	18	54.92043	86.32	28589.96
1999Q2	4772.3	98394.12	216085.5	18	74.19829	93.25	28406.38
1999Q3	5032.1	98546.73	231512.1	18	82.79302	94.88	28222.79
1999Q4	5424.6	98066.84	245697.7	18	90.22541	96.32	28039.21
2000Q1	6682.8	103201.2	160731.3	13.5	79.42832	99.88	28097.83
2000Q2	7272.4	103182.9	173001.3	13.5	84.18086	101.12	28156.45
2000Q3	8118.1	103234.4	177015.1	13.5	74.00978	103.53	28215.06
2000Q4	9386.1	102713.5	182873.3	13.5	68.93036	103.9	28273.68
2001Q1	10787.5	108099.8	311394.2	14.3	75.85664	110.62	28292.01
2001Q2	10166.7	108093.2	348097.9	14.3	76.97549	113.25	28310.34
2001Q3	10563.9	108083.7	364182.3	14.3	73.9196	111.71	28328.67
2001Q4	10267.1	107506.5	323791.8	14.3	60.18938	112.19	28347
2002Q1	9546.1	112633	376098.8	19	46.14942	114.76	29008.22
2002Q2	8674.7	113328.2	237339.6	19	39.70769	117.06	29669.44
2002Q3	7424	113096.1	325514.1	19	54.00043	125.31	30330.65
2002Q4	7681.1	112728.4	328984	19	59.65409	126.76	30991.87
2003Q1	8226.16	124036.8	543697.5	15.75	64.60292	127.18	31473.11
2003Q2	7673.09	123928.7	637173.7	15.75	64.1783	127.62	31954.34
2003Q3	7170.46	123782.6	587857.7	15.75	64.74057	128.08	32435.58
2003Q4	7467.78	123259	592867.6	15.75	69.97847	134.54	32916.81
2004Q1	9684.49	114617.6	542251	15	58.83204	135.23	33673.77
2004Q2	11441.36	123702.9	542199.2	15	63.4855	133.09	34430.74
2004Q3	13222.9	142373.6	548465.9	15	64.96385	132.82	35187.7
2004Q4	16955.02	146881.9	561051	15	69.51445	132.86	35944.66
2005Q1	21807.98	120048.9	606281.3	13	64.53038	132.85	32077.99
2005Q2	24367.12	128755.5	620972.6	13	63.33277	132.85	28211.32
2005Q3	28638.24	153933.6	631451.6	13	62.4192	132.3	24344.64
2005Q4	28279.06	159193.4	637718.2	13	60.55284	130.59	20477.97
2006Q1	36201.54	128579.8	639120.7	12.3	66.7117	129.53	16244.6
2006Q2	36479	135438.6	612365	12.3	53.46805	128.46	12011.23
2006Q3	40457.86	162498.8	839306.6	12.3	58.49019	128.33	7777.86
2006Q4	42298.11	169304.4	719713.3	12.3	46.90873	128.29	3544.49
2007Q1	42633.86	135774.9	1116317	8.8	58.42221	128.23	3287.73
2007Q2	42626.2	142763.5	1072817	8.8	55.67136	127.65	3348.22
2007Q3	47930.22	173067.5	1336873	8.8	57.94592	126.58	3397.48

2007Q4	51333.15	182618.2	813760.5	8.8	46.01118	120.87	3654.21
2008Q1	59756.51	142071.4	824101.6	9.8	60.34545	118.04	3670.748
2008Q2	59157.15	150862.2	857801.1	9.8	62.17349	117.84	3687.285
2008Q3	62081.86	183678.8	838843.8	9.8	52.98382	117.75	3703.823
2008Q4	53000.36	195590.1	778350.1	9.8	38.72028	120.65	3720.36
2009Q1	47081.9	149191.5	957725.1	7.4	36.53433	146.88	3719.8
2009Q2	43462.74	162101.2	994515.1	7.4	37.58924	147.76	3719.24
2009Q3	43343.33	197084.3	1310612	7.4	52.8128	150.92	3863.93
2009Q4	42382.49	210600.4	1785016	7.4	70.66926	149.96	3947.3
2010Q1	40667.03	160117	1453138	6.1	0.794283	149.8285	4306.18
2010Q2	37468.44	174734	1332006	6.1	0.841809	150.1915	4269.71
2010Q3	34589.01	212771.7	1668885	6.1	0.740098	151.0332	4534.19
2010Q4	32339.25	228709.5	1469228	6.1	0.689304	150.4799	4578.77
2011Q1	33221.8	171265.9	1672986	9.2	0.758566	152.5074	5227.05
2011Q2	31890.91	187833.1	2476227	9.2	0.769755	154.5029	5398.04
2011Q3	31740.23	228454.8	2276061	9.2	0.739196	155.2636	5633.71
2011Q4	32639.78	246447.1	2948707	9.2	0.62	158.2074	5666.58
2012Q1	35197.44	182119.4	2253505	12	0.58	157.5875	5993.54
2012Q2	35412.5	199831.6	2962151	12	0.56	157.4383	6035.66
2012Q3	40640.4	243263.1	2879259	12	0.52	157.3429	6296.17
2012Q4	43830.42	263678.9	3810044	12	0.53	157.324	6527.07

Appendix 2

$$M_{n\times m} = \begin{bmatrix} 0 & 0 & 0 & 0 & t_{115} & m_{116} & e_{117} & 0 & 1 \\ r_{121} & 0 & g_{123} & 0 & t_{125} & m_{126} & e_{127} & s_{12,m-1} & 1 \\ r_{131} & r_{232} & g_{133} & 0 & t_{135} & m_{136} & e_{137} & s_{13,m-1} & 1 \\ r_{141} & r_{242} & g_{143} & g_{244} & t_{145} & m_{146} & e_{147} & s_{14,m-1} & 1 \\ r_{151} & r_{252} & g_{153} & g_{254} & t_{155} & m_{156} & e_{157} & s_{15,m-1} & 1 \\ r_{161} & r_{262} & g_{163} & g_{264} & t_{165} & m_{166} & e_{167} & s_{16,m-1} & 1 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ r_{1n-1,1} & r_{2n-1,2} & g_{1n-1,3} & g_{2n-1,4} & t_{1n-1,5} & m_{1n-1,6} & e_{1n-1,7} & s_{1n-1,m-1} & 1 \\ r_{1n,1} & r_{2n,2} & g_{1n,3} & g_{2n,4} & t_{1n,5} & m_{1n,6} & e_{1n,7} & s_{1n,m-1} & 1 \end{bmatrix}$$

$$M^T_{m\times n} = \begin{bmatrix} 0 & r_{112} & r_{113} & r_{114} & r_{115} & r_{116} & . & . & . & r_{11n-1} & r_{11n} \\ 0 & 0 & r_{223} & r_{224} & r_{225} & r_{226} & . & . & . & r_{22n-1} & r_{22n} \\ 0 & g_{132} & g_{133} & g_{134} & g_{135} & g_{136} & . & . & . & g_{13n-1} & g_{13n} \\ 0 & 0 & 0 & g_{244} & g_{245} & g_{246} & . & . & . & g_{24n-1} & g_{14n} \\ t_{151} & t_{152} & t_{153} & t_{154} & t_{155} & t_{156} & . & . & . & t_{15n-1} & t_{15n} \\ m_{161} & m_{162} & m_{163} & m_{164} & m_{165} & m_{166} & . & . & . & m_{16n-1} & m_{16n} \\ e_{171} & e_{172} & e_{173} & e_{174} & e_{175} & e_{176} & . & . & . & e_{17n-1} & e_{17n} \\ 0 & s_{1m-1,2} & s_{1m-1,3} & s_{1m-1,4} & s_{1m-1,5} & s_{1m-1,6} & . & . & . & s_{1m-1,n-1} & s_{1m-1,n} \\ 1 & 1 & 1 & 1 & 1 & 1 & . & . & . & 1 & 1 \end{bmatrix}$$

$$LR = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ . \\ . \\ . \\ r_n \end{bmatrix}, \qquad ɣ = \begin{bmatrix} ɣ_1 \\ ɣ_2 \\ ɣ_3 \\ ɣ_4 \\ ɣ_5 \\ . \\ . \\ . \\ ɣ_m \end{bmatrix}$$

$$Z_{m\times m} = \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & z_{1,m-2} & z_{1,m-1} & z_{1m} \\ z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} & z_{2,m-2} & z_{2,m-1} & z_{2m} \\ z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} & z_{3,m-2} & z_{3,m-1} & z_{3m} \\ z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} & z_{4,m-2} & z_{4,m-1} & z_{4m} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{56} & z_{5,m-2} & z_{5,m-1} & z_{5m} \\ z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} & z_{6,m-2} & z_{6,m-1} & z_{6m} \\ z_{m-2,1} & z_{m-2,2} & z_{m-2,3} & z_{m-2,4} & z_{m-2,5} & z_{m-2,6} & z_{m-2,m-2} & z_{m-2,m-1} & z_{m-2,m} \\ z_{m-1,1} & z_{m-1,2} & z_{m-1,3} & z_{m-1,4} & z_{m-1,5} & z_{m-1,6} & z_{m-1,m-2} & z_{m-1,m-1} & z_{m-1,m} \\ z_{m1} & z_{m2} & z_{m3} & z_{m4} & z_{m5} & z_{m6} & z_{m,m-2} & z_{m,m-1} & z_{mm} \end{bmatrix}$$

$$H_{(m\times 1)} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ . \\ . \\ . \\ h_m \end{bmatrix}$$

$$ɣ = \begin{bmatrix} ɣ_1 \\ ɣ_2 \\ ɣ_3 \\ ɣ_4 \\ ɣ_5 \\ ɣ_6 \\ . \\ . \\ . \\ ɣ_m \end{bmatrix}$$

$$\dot{M}_{n\times(m-1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & t_{115} & e_{116} & 0 & 1 \\ r_{121} & 0 & g_{123} & 0 & t_{125} & e_{126} & s_{12,m-1} & 1 \\ r_{131} & r_{232} & g_{133} & 0 & t_{135} & e_{136} & s_{13,m-1} & 1 \\ r_{141} & r_{242} & g_{143} & g_{244} & t_{145} & e_{146} & s_{14,m-1} & 1 \\ r_{151} & r_{252} & g_{153} & g_{254} & t_{155} & e_{156} & s_{15,m-1} & 1 \\ r_{161} & r_{262} & g_{163} & g_{264} & t_{165} & e_{166} & s_{16,m-1} & 1 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ r_{1n-1,1} & r_{2n-1,2} & g_{1n-1,3} & g_{2n-1,4} & t_{1n-1,5} & e_{1n-1,6} & s_{1n-1,m-1} & 1 \\ r_{1n,1} & r_{2n,2} & g_{1n,3} & g_{2n,4} & t_{1n,5} & e_{1n,6} & s_{1n,m-1} & 1 \end{bmatrix}$$

$$\dot{M}^T_{(m-1)\times n} = \begin{bmatrix} 0 & r_{112} & r_{113} & r_{114} & r_{115} & r_{116} & . & . & . & r_{11n-1} & r_{11n} \\ 0 & 0 & r_{223} & r_{224} & r_{225} & r_{226} & . & . & . & r_{22n-1} & r_{22n} \\ 0 & g_{132} & g_{133} & g_{134} & g_{135} & g_{136} & . & . & . & g_{13n-1} & g_{13n} \\ 0 & 0 & 0 & g_{244} & g_{245} & g_{246} & . & . & . & g_{24n-1} & g_{14n} \\ t_{151} & t_{152} & t_{153} & t_{154} & t_{155} & t_{156} & . & . & . & t_{15n-1} & t_{15n} \\ e_{161} & e_{162} & e_{163} & e_{164} & e_{165} & e_{166} & . & . & . & e_{16n-1} & e_{16n} \\ 0 & s_{1m-1,2} & s_{1m-1,3} & s_{1m-1,4} & s_{1m-1,5} & s_{1m-1,6} & . & . & . & s_{1m-1,n-1} & s_{1m-1,n} \\ 1 & 1 & 1 & 1 & 1 & 1 & . & . & . & 1 & 1 \end{bmatrix}$$

$$LR_{(n\times 1)} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ . \\ . \\ . \\ r_n \end{bmatrix},$$

$$\dot{\gamma} = \begin{bmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_2 \\ \dot{\gamma}_3 \\ \dot{\gamma}_4 \\ \dot{\gamma}_5 \\ . \\ . \\ . \\ \dot{\gamma}_{m-1} \end{bmatrix}$$

$$\dot{Z}_{(m-1)\times(m-1)} = \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{1,m-1} \\ z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{2,m-1} \\ z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{3,m-1} \\ z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{4,m-1} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{5,m-1} \\ z_{m-3,1} & z_{m-3,2} & z_{m-3,3} & z_{m-3,4} & z_{m-3,5} & z_{m-3,m-1} \\ z_{m-2,1} & z_{m-2,2} & z_{m-2,3} & z_{m-2,4} & z_{m-2,5} & z_{m-2,m-1} \\ z_{m-1,1} & z_{m-1,2} & z_{m-1,3} & z_{m-1,4} & z_{m-1,5} & z_{m-1,m-1} \end{bmatrix}$$

$$\dot{H}_{(m-1)\times 1} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ . \\ . \\ . \\ h_{m-1} \end{bmatrix}$$

$$\dot{\gamma} = \begin{bmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_2 \\ \dot{\gamma}_3 \\ \dot{\gamma}_4 \\ \dot{\gamma}_5 \\ . \\ . \\ . \\ \dot{\gamma}_{m-1} \end{bmatrix}$$

$$\ddot{M}_{n\times(m-2)} = \begin{bmatrix} 0 & 0 & 0 & 0 & t_{115} & e_{116} & 1 \\ r_{121} & 0 & g_{123} & 0 & t_{125} & e_{126} & 1 \\ r_{131} & r_{232} & g_{133} & 0 & t_{135} & e_{136} & 1 \\ r_{141} & r_{242} & g_{143} & g_{244} & t_{145} & e_{146} & 1 \\ r_{151} & r_{252} & g_{153} & g_{254} & t_{155} & e_{156} & 1 \\ r_{161} & r_{262} & g_{163} & g_{264} & t_{165} & e_{166} & 1 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ r_{1n-1,1} & r_{2n-1,2} & g_{1n-1,3} & g_{2n-1,4} & t_{1n-1,5} & e_{1n-1,6} & 1 \\ r_{1n,1} & r_{2n,2} & g_{1n,3} & g_{2n,4} & t_{1n,5} & e_{1n,6} & 1 \end{bmatrix}$$

$$\ddot{M}^{T}_{(m-2)\times n} = \begin{bmatrix} 0 & r_{112} & r_{113} & r_{114} & r_{115} & r_{116} & . & . & . & r_{11n-1} & r_{11n} \\ 0 & 0 & r_{223} & r_{224} & r_{225} & r_{226} & . & . & . & r_{22n-1} & r_{22n} \\ 0 & g_{132} & g_{133} & g_{134} & g_{135} & g_{136} & . & . & . & g_{13n-1} & g_{13n} \\ 0 & 0 & 0 & g_{244} & g_{245} & g_{246} & . & . & . & g_{24n-1} & g_{14n} \\ t_{151} & t_{152} & t_{153} & t_{154} & t_{155} & t_{156} & . & . & . & t_{15n-1} & t_{15n} \\ e_{161} & e_{162} & e_{163} & e_{164} & e_{165} & e_{166} & . & . & . & e_{16n-1} & e_{16n} \\ 1 & 1 & 1 & 1 & 1 & 1 & . & . & . & 1 & 1 \end{bmatrix}$$

$$LR_{(n\times 1)} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ . \\ . \\ . \\ r_n \end{bmatrix},$$

$$\ddot{\gamma} = \begin{bmatrix} \ddot{\gamma}_1 \\ \ddot{\gamma}_2 \\ \ddot{\gamma}_3 \\ \ddot{\gamma}_4 \\ \ddot{\gamma}_5 \\ . \\ . \\ . \\ \ddot{\gamma}_{m-2} \end{bmatrix}$$

$$\ddot{Z}_{(m-2)\times(m-2)} = \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & . & . & . & z_{1,m-2} \\ z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} & . & . & . & z_{2,m-2} \\ z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} & . & . & . & z_{3,m-2} \\ z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} & . & . & . & z_{4,m-2} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{56} & . & . & . & z_{5,m-2} \\ z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} & . & . & . & z_{6,m-2} \\ z_{m-3,1} & z_{m-3,2} & z_{m-3,3} & z_{m-3,4} & z_{m-3,5} & z_{m-3,6} & . & . & . & z_{m-3,m-2} \\ z_{m-2,1} & z_{m-2,2} & z_{m-2,3} & z_{m-2,4} & z_{m-2,5} & z_{m-2,6} & . & . & . & z_{m-2,m-2} \end{bmatrix}$$

$$\ddot{H}_{(m-2)\times 1} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ . \\ . \\ . \\ h_{m-2} \end{bmatrix}$$

$$\ddot{\gamma} = \begin{bmatrix} \ddot{\gamma}_1 \\ \ddot{\gamma}_2 \\ \ddot{\gamma}_3 \\ \ddot{\gamma}_4 \\ \ddot{\gamma}_5 \\ . \\ . \\ . \\ \ddot{\gamma}_{m-2} \end{bmatrix}$$

$$
\ddot{M}_{n\times(m-3)} = \begin{bmatrix}
0 & 0 & 0 & 0 & t_{115} & 1 \\
r_{121} & 0 & g_{123} & 0 & t_{125} & 1 \\
r_{131} & r_{232} & g_{133} & 0 & t_{135} & 1 \\
r_{141} & r_{242} & g_{143} & g_{244} & t_{145} & 1 \\
r_{151} & r_{252} & g_{153} & g_{254} & t_{155} & 1 \\
r_{161} & r_{262} & g_{163} & g_{264} & t_{165} & 1 \\
. & . & . & . & . & . \\
. & . & . & . & . & . \\
. & . & . & . & . & . \\
r_{1n-1,1} & r_{2n-1,2} & g_{1n-1,3} & g_{2n-1,4} & t_{1n-1,5} & 1 \\
r_{1n,1} & r_{2n,2} & g_{1n,3} & g_{2n,4} & t_{1n,5} & 1
\end{bmatrix}
$$

$$
\ddot{M}^{T}_{(m-3)\times n} = \begin{bmatrix}
0 & r_{112} & r_{113} & r_{114} & r_{115} & r_{116} & . & . & . & r_{11n-1} & r_{11n} \\
0 & 0 & r_{223} & r_{224} & r_{225} & r_{226} & . & . & . & r_{22n-1} & r_{22n} \\
0 & g_{132} & g_{133} & g_{134} & g_{135} & g_{136} & . & . & . & g_{13n-1} & g_{13n} \\
0 & 0 & 0 & g_{244} & g_{245} & g_{246} & . & . & . & g_{24n-1} & g_{14n} \\
t_{151} & t_{152} & t_{153} & t_{154} & t_{155} & t_{156} & . & . & . & t_{15n-1} & t_{15n} \\
1 & 1 & 1 & 1 & 1 & 1 & . & . & . & 1 & 1
\end{bmatrix}
$$

$$
LR_{(n\times1)} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ . \\ . \\ . \\ r_n \end{bmatrix},
$$

$$
\ddot{Y} = \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \\ \ddot{y}_4 \\ \ddot{y}_5 \\ . \\ . \\ . \\ \ddot{y}_{m-3} \end{bmatrix}
$$

$$
\ddot{Z}_{(m-3)\times(m-3)} = \begin{bmatrix}
z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & . & . & . & z_{1,m-3} \\
z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} & . & . & . & z_{2,m-3} \\
z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} & . & . & . & z_{3,m-3} \\
z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} & . & . & . & z_{4,m-3} \\
z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{56} & . & . & . & z_{5,m-3} \\
z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} & . & . & . & z_{6,m-3} \\
z_{m-3,1} & z_{m-3,2} & z_{m-3,3} & z_{m-3,4} & z_{m-3,5} & z_{m-3,6} & . & . & . & z_{m-3,m-3}
\end{bmatrix}
$$

$$
\ddot{H}_{(m-3)\times1} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ . \\ . \\ . \\ h_{m-3} \end{bmatrix}
$$

$$
\ddot{y} = \ddot{Y}_{(m-3)\times1}, \quad \ddot{y}^{T} = [\ddot{y}_1 \quad \ddot{y}_2 \quad \ddot{y}_3 \quad \ddot{y}_4 \quad \ddot{y}_5 \quad . \quad . \quad . \quad \ddot{y}_{m-3}]
$$

Appendix 3

The matrix $M_{n \times m}$ must be stable in its parameter values since we are considering the least parsimonious model.

Table 1: The least Parsimonious Model for Long – run Parameters
Dependent Variable: LR
Sample: 2000Q1 2012Q4
Included observations: 52

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-11.03724	2.429650	-4.542728	0.0000
LR(-1)	1.180918	0.130865	9.023968	0.0000
LR(-2)	-0.344818	0.134846	-2.557121	0.0142
LG(-1)	0.658327	0.155491	4.233864	0.0001
LG(-3)	0.657772	0.158750	4.143449	0.0002
LT	0.053448	0.013895	3.846539	0.0004
LM	-0.144597	0.076465	-1.891015	0.0654
LE	-0.781727	0.255538	-3.059146	0.0038
LS(-1)	0.114542	0.044735	2.560432	0.0140

R-squared	0.990537	Mean dependent var	10.01357
Adjusted R-squared	0.988777	S.D. dependent var	0.749769
S.E. of regression	0.079431	Akaike info criterion	-2.071754
Sum squared resid	0.271297	Schwarz criterion	-1.734039
Log likelihood	62.86561	Hannan-Quinn criter.	-1.942282
F-statistic	562.6389	Durbin-Watson stat	1.964195
Prob(F-statistic)	0.000000		

Table 2: Error Correction Representation for the Selected ARDL Model (Short-run)
ARDL (3, 3, 0, 0, 1) selected based on Schwarz Bayesian Criterion

```
******************************************************************************
 Dependent variable is dLR
 52 observations used for estimation from 2000Q1 to 2012Q4

******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio [Prob]
 dLR1                       .34197              .13630            2.5089[.016]
 dLR2                       .33445              .13373            2.5009[.016]
 dLG                       -.20034              .15538           -1.2894[.204]
 dLG1                      -.41951              .17266           -2.4296[.019]
 dLG2                      -.53121              .16392           -3.2406[.002]
 dLT                       .044387             .014414            3.0795[.004]
 dLM                      -.058236             .060824           -.95744[.344]
 dLE                       -.94057              .40824           -2.3039[.026]
 dINPT                     -6.6001              2.1323           -3.0953[.003]
 ecm(-1)                   -.19126             .053827           -3.5532[.001]
******************************************************************************
R-Squared                     .58711   R-Bar-Squared                    .47357
 S.E. of Regression          .077199   F-stat.    F (9, 42)     6.3198[.000]
 Mean of Dependent Variable  .040180   S.D. of Dependent Variable       .10640
 Residual Sum of Squares      .23839   Equation Log-likelihood         66.2277
 Akaike Info. Criterion      54.2277   Schwarz Bayesian Criterion      42.5203
 DW-statistic                 2.0923
******************************************************************************
```

Table 3: Revised version of Irefin's and Yaaba's model (including trade openness and external debt) with CUSUM/CUSUMSQ

Dependent Variable: LR
Date: 10/07/14 Time: 14:57
Sample: 1999Q1 2011Q2
Included observations: 50

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-9.456340	3.079640	-3.070599	0.0038
LR(-1)	1.337251	0.125600	10.64692	0.0000
LR(-2)	-0.502723	0.135358	-3.714019	0.0006
LG(-1)	0.519842	0.183180	2.837879	0.0070
LG(-3)	0.523464	0.195021	2.684150	0.0104
LT	0.056753	0.016247	3.493193	0.0012
LM	-0.153850	0.106678	-1.442183	0.1568
LE	-0.366501	0.233858	-1.567195	0.1248
LS(-1)	0.077155	0.048987	1.575018	0.1229

R-squared	0.990445	Mean dependent var	9.838195
Adjusted R-squared	0.988581	S.D. dependent var	0.835348
S.E. of regression	0.089265	Akaike info criterion	-1.832869
Sum squared resid	0.326697	Schwarz criterion	-1.488705
Log likelihood	54.82172	Hannan-Quinn criter.	-1.701809
F-statistic	531.2647	Durbin-Watson stat	2.129299
Prob(F-statistic)	0.000000		

Table 4: Error Correction Representation for the Selected ARDL Model

ARDL (3, 3, 0, 0, 1, 1) selected based on Schwarz Bayesian Criterion

Dependent variable is dLR
50 observations used for estimation from 1999Q1 to 2011Q2

Regressor	Coefficient	Standard Error	T-Ratio [Prob]
dLR1	.35836	.13626	2.6300[.012]
dLR2	.25833	.14442	1.7887[.081]
dLG	-.23281	.18181	-1.2805[.208]
dLG1	-.64165	.20948	-3.0631[.004]
dLG2	-.73639	.20934	-3.5177[.001]
dLT	.052277	.015401	3.3944[.002]
dLM	-.13927	.098342	-1.4162[.165]
dLE	-.65849	.26984	-2.4403[.019]
dLS	-.15022	.099817	-1.5049[.140]
dINPT	-8.8673	2.9692	-2.9864[.005]
ecm(-1)	-.22712	.068553	-3.3131[.002]

R-Squared	.66769	R-Bar-Squared	.54769
S.E. of Regression	.079174	F-stat. F (10, 39)	7.2333[.000]
Mean of Dependent Variable	.030023	S.D. of Dependent Variable	.11772
Residual Sum of Squares	.22567	Equation Log-likelihood	64.0710
Akaike Info. Criterion	50.0710	Schwarz Bayesian Criterion	36.6868
DW-statistic	2.0878		

Stability Test (CUSUM and CUSUMSQ) Sample 2000Q1 2012Q4

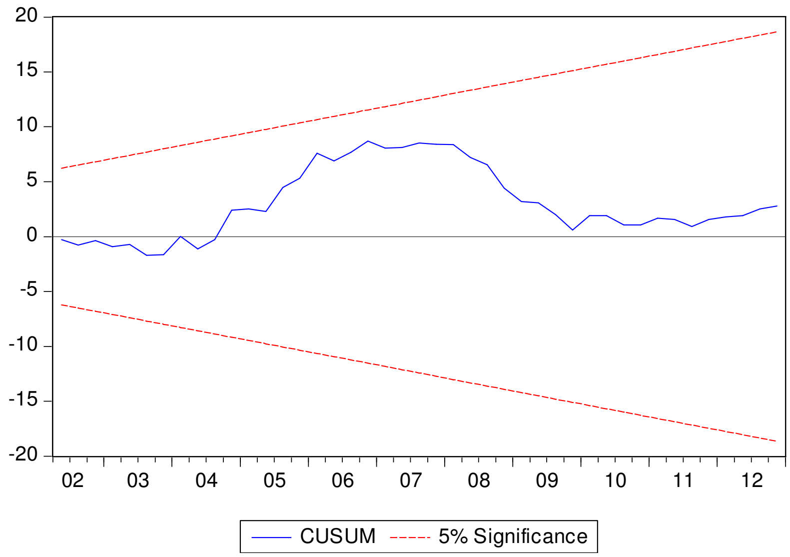

Figure 1: Stability of the Least Parsimonious Model with cumulative sum of recursive residuals at 5%

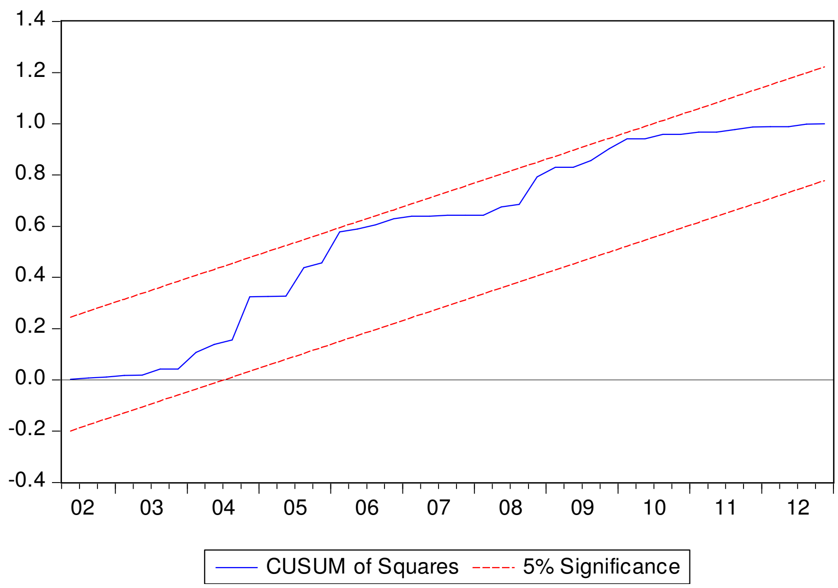

Figure 2: Stability of the Least Parsimonious Model with cumulative sum of squares of recursive residuals at 5%.

Stability Test (CUSUM and CUSUMSQ) Sample 1999Q1 2011Q2

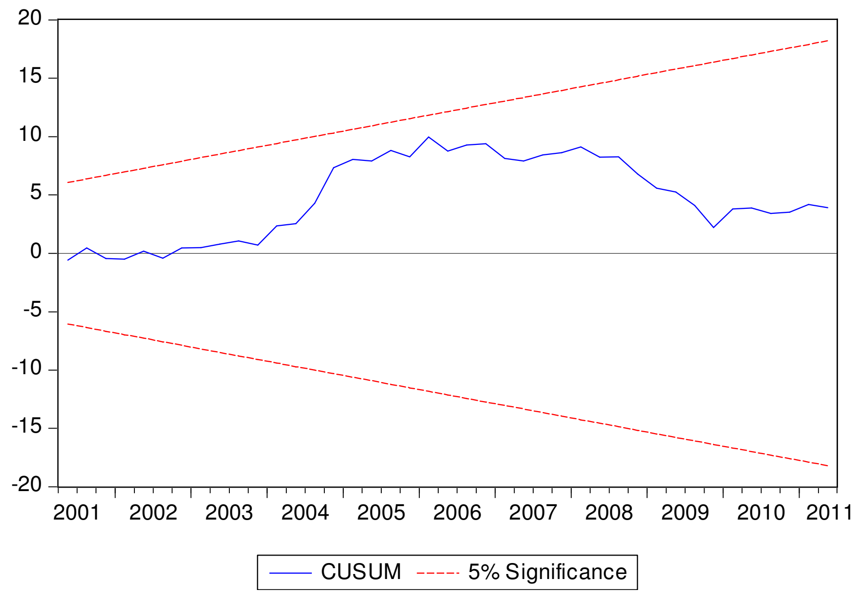

Figure 3: Stability of the Least Parsimonious Model by cumulative sum at 5%.

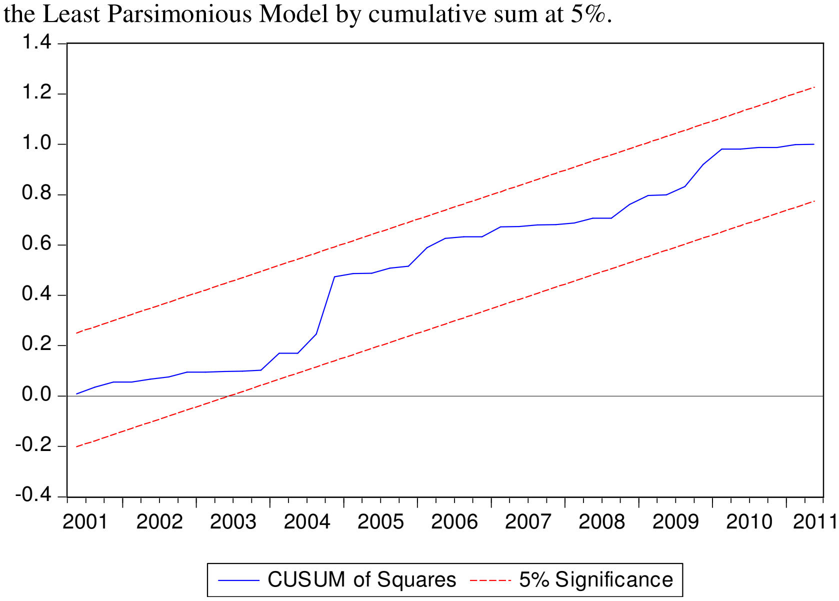

Figure 4: Stability of the Least Parsimonious Model by cumulative sum of squares of recursive residuals at 5%.

3.4 Analysis of the Binomial Model and Modified ARDL Model for different order of n(k)

Table 5: Data Period: 2000Q1-2012Q4

r \ n(K)	3	4	5	6	7	8	9
1	NS	NS	NS	NS	NS	NS	NS
2	1, **1**	1,**1**	1,**1**	1,**1**	1,**1**	1,**1**	1, **1**
3	1, **NS**	2, **NS**	3,**2**	4,**4**	5,**5**	6,**6**	7, **7**
4	NA	1,**1**	3,**2**	6,**7**	10,**11**	15,**17**	21, **21**
5	NA	NA	1,**1**	4,**4**	10, **13**	20, **22**	35, **35**
6	NA	NA	NA	1,**1**	5,**5**	15,**16**	35, **37**
7	NA	NA	NA	NA	1,**1**	6,**7**	21, **24**
8	NA	NA	NA	NA	NA	1,**1**	7, **8**
9	NA	NA	NA	NA	NA	NA	1,**1**

Table 6: Data Period: 1999Q1-2011Q2

r \ n(K)	3	4	5	6	7	8	9
1	NS	NS	NS	NS	NS	NS	NS
2	1, **1**	1,**1**	1,**1**	1,**1**	1,**1**	1,**1**	1, **1**
3	1, **NS**	2, **2**	3,**4**	4,**5**	5,**5**	6,**7**	7, **7**
4	NA	1,**1**	3,**4**	6,**9**	10,**12**	15,**18**	21, **20**
5	NA	NA	1,**1**	4,**5**	10, **9**	20, **20**	35, **26**
6	NA	NA	NA	1,**1**	5,**6**	15,**17**	35, **31**
7	NA	NA	NA	NA	1,**1**	6,**6**	21, **21**
8	NA	NA	NA	NA	NA	1,**1**	7, **7**
9	NA	NA	NA	NA	NA	NA	1,**1**

Where Bold face is $n(S_r(M))$ = Stability Results from EView, while faces not bolded is $n(S_r(B))$ = Stability from the respective families of the Binomial coefficients. NS means Not Stable and NA means Not Applicable.

Appendix 4

$$M_{n\times m}=\begin{bmatrix} 0 & 0 & 0 & 0 & 4.374855 & 2.60269 & 4.603969 & 0 & 1 \\ 8.89184 & 0 & 11.54426 & 0 & 4.432968 & 2.60269 & 4.616308 & 10.24553 & 1 \\ 9.00185 & 9.00185 & 11.54476 & 0 & 4.304197 & 2.60269 & 4.639861 & 10.24761 & 1 \\ 9.14699 & 9.14699 & 11.5397 & 11.5397 & 4.233097 & 2.60269 & 4.643429 & 10.24969 & 1 \\ 9.28614 & 9.28614 & 11.59081 & 11.59081 & 4.328845 & 2.66026 & 4.706101 & 10.25033 & 1 \\ 9.22687 & 9.22687 & 11.59075 & 11.59075 & 4.343487 & 2.66026 & 4.729598 & 10.25098 & 1 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 10.6125 & 10.6125 & 12.4019 & 12.4019 & -0.65393 & 2.484907 & 5.058428 & 8.747697 & 1 \\ 10.6881 & 10.6881 & 12.48249 & 12.48249 & -0.63488 & 2.484907 & 5.058307 & 8.783713 & 1 \end{bmatrix}$$

$$M^{T}_{m\times n}=\begin{bmatrix} 0 & 8.89184 & 9.00185 & 9.14699 & 9.28614 & 9.22687 & . \; . \; . & 10.6125 & 10.6881 \\ 0 & 0 & 9.00185 & 9.14699 & 9.28614 & 9.22687 & . \; . \; . & 10.6125 & 10.6881 \\ 0 & 11.54426 & 11.54476 & 11.5397 & 11.59081 & 11.59075 & . \; . \; . & 12.4019 & 12.48249 \\ 0 & 0 & 0 & 11.5397 & 11.59081 & 11.59075 & . \; . \; . & 12.4019 & 12.48249 \\ 4.374855 & 4.432968 & 4.304197 & 4.233097 & 4.328845 & 4.343487 & . \; . \; . & -0.65393 & -0.63488 \\ 2.60269 & 2.60269 & 2.60269 & 2.60269 & 2.66026 & 2.66026 & . \; . \; . & 2.484907 & 2.484907 \\ 4.603969 & 4.616308 & 4.639861 & 4.643429 & 4.706101 & 4.729598 & . \; . \; . & 5.058428 & 5.058307 \\ 0 & 10.24553 & 10.24761 & 10.24969 & 10.25033 & 10.25098 & . \; . \; . & 8.747697 & 8.783713 \\ 1 & 1 & 1 & 1 & 1 & 1 & . \; . \; . & 1 & 1 \end{bmatrix}$$

$$LR^{T} = [8.80729 \quad 8.89184 \quad 9.00185 \quad 9.14699 \quad 9.28614 \quad 9.22687 \quad . \; . \; . \quad 10.6125 \quad 10.6881]$$

$$\gamma=\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \gamma_5 \\ . \\ . \\ . \\ \gamma_m \end{bmatrix}$$

Thus,

$$M^{T}_{m\times n}M_{n\times m}\gamma = M^{T}_{m\times n}\, LR$$