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ON THE PREDICTION OF NON-HOMOGENEOUS MARKOV FUZZY MANPOWER SYSTEMS

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ABSTRACT

Many research works have considered intra-state heterogeneity in personnel transition behaviour due to latent factors. None of these research works has captured any specific latent factor or combination of specific latent factors which influence the within-state transition differences in a manpower model. Therefore, this work considers a hierarchical non-homogeneous manpower system in which promotion of employees is only based on the level of innovativeness and job performance capability of the employees. In this system, a non-homogeneous Markov model which takes into account theory of fuzzy sets is proposed to deal with differences in transition behaviour of members belonging to the same category which in theory are assumed equal for every member of same category and to incorporate key personality traits which influence employees within a homogeneous category to behave in different ways. The total transition probability matrix is estimated. The limiting probability structure for the fuzzy manpower system is obtained as $[0.2181, 0.2233, 0.20280, 0.3505]$. *This result suggests that greater proportion of staff would possess advance level of openness and advance level of conscientiousness in the long run compared to other levels of combinations of personnel traits.*

KEYWORDS: *Fuzzy Set, Fuzzy state, Fuzzy Manpower System, Membership Function, membership degree, Non-Homogeneous Markov Chain*

INTRODUCTION

Fuzzy set is a set that does not have clearly defined boundaries (limits) and can contain members only at some degree. A fuzzy set D on a nonempty set U is defined mathematically as a set of ordered pairs$\{(U, \mu_D(u), u \in U)\}$, where $\mu_D(u)$ denotes the membership grade of u in D. The set D is characterised by its membership function $\mu_D: U \to [0,1]$. The basic idea of fuzzy sets is concerned with its flexibility over the concept of membership (belongingness). Although many sets have sharp boundaries (eg sex and marital status), there are several situations (eg manpower system's classification) where membership relation is not clearly defined. For this reason, (Zadeh, 1968), proposed a membership degree according to which a member can belong to the set partially. Fuzzy set theory is an extension of classical set theory proposed by (Zadeh, 1968), that provides a mathematical framework for handling categories that permit partial membership (or membership in degree). The fuzzy state in a Non-Homogeneous Markov System (NHMS) is defined mathematically by assigning to each possible member of the state a value representing its grade of membership in the fuzzy state (Symeonaki *et al*., 2002). Thus, fuzzy manpower system is a manpower system that consists of fuzzy states. Therefore, Non-Homogeneous Markov fuzzy manpower system (NHMFMS) is a manpower system whose transitions of members are assumed to be dependent on time and whose states are fuzzy.

Manpower system consists of personnel working together for the purpose of achieving the common goal of an organization. Human Resources Planning (HRP) represents the range of philosophies, tools and techniques that any organization should deploy to monitor and manage the movement of staff, both in terms of numbers and profiles, (Behlaji and Tkiouat 2013). Manpower planning is concerned with personnel supply-and-demand prediction and development of personnel strategy which ensures that the required personnel are available at the right time, (Bartholomew *et al.,* 1991). In aggregate, the workforce system of any organisation comprises of a stock of heterogeneous personnel. In most manpower models, such as Ezugwu and Igbinosun (2020) and Vassiliou (2021), the manpower system is hierarchically graded into mutually exclusive and exhaustive grades so that each member of the system may belong to one and only one of the grades at any given time. The aggregated personnel system is partitioned into homogeneous groups in such a way that members of staff in the same grade have certain common attributes and are presumed to evolve analogously, (De Feyter, 2006). A popular mathematical model for modelling manpower systems is Markovian model. Markov chain theory is widely used in manpower planning for forecasting, as well as control of personnel structure, (Symeonaki and Stamou, 2004, Jeeva and Geetha, 2013). Markov chain theory is also used in portfolio allocation and market equilibrium mix; (Ezugwu *et al.*, 2013; Ezugwu and Igbinosun, 2016). For these manpower models, the aggregated personnel systems are classified into homogeneous groups on the basis of whatever attributes that are relevant for the problem at hand, and it is presumed that each group satisfies the central assumption of Markov theory that every member of a particular grade has equal rate of transition to the next higher grade and that the probability of moving to the next state is dependent on the current state not on the previous states. Concerning the Markovian approach for modelling manpower systems, Markov models can be classified into homogeneous or non-homogeneous, based on the nature of the system's dependency on time. A Markov chain model is said to be homogeneous if the transition probabilities of the members are assumed to be independent of time (that is, they do not change over time). Examples of such models include; (Ekhosuehi and Osagiede, 2006; Ekhosuehi *et al.* 2017; Ezugwu and Ologun 2017; Ezugwu and Igbinosun 2020).

A Markov chain model is said to be non-homogeneous if the transition probabilities of the members are assumed to be dependent on time. Examples include (Vassiliou, 2021; Vassiliou, 2022). In this study, the concept of non-homogeneous Markov manpower system is considered. The concept of Non-homogeneous Markov systems was first introduced in (Vassiliou, 1982) and the motive was to provide a more general framework for several Non-homogeneous Markov chain models in manpower systems (Vassiliou, 2018). In this study, we examine the necessity of introducing fuzzy states in non-homogeneous Markov manpower systems. Like it was earlier stressed, manpower planning analysis based on Markovian approach assumes that the system of interest is partitioned into distinct classes (states), where each member of the system "clearly" belongs to one and only one of the classes at time t, and makes transition from one states to another at time $t+1$. In other word, Markovian approach requires that the states of the system under study be precisely measured and defined in such a way that the members of the system are dichotomized into two groups: members and non-members. However, this assumption is unrealistic in some situations regarding manpower systems' classification. In some situations, in real application of Markov theory in manpower planning analysis, one is often faced with the fact of fuzzy states, in the sense that the states of the system cannot be precisely measured due to vagueness in transition of members from particular states to another. This method of classification of states of Markov system into fuzzy states can also be appropriate for manpower systems. For instance, different personnel belonging to a certain grade do exhibit different rates of transition to the next higher grades of a hierarchical manpower system. In hierarchical manpower system, promotion of a member of the system from current grade to the next higher grade is normally possible after the member has completed all the necessary requirements/conditions for promotion, peculiar to that particular origin state. It is realistic that at a particular time t, different members of the same grade have different current levels of completion of promotion requirements peculiar to the state, which guarantee their next promotion to the next higher grade at different times $t+1, t+2, \ldots\ldots$ It may, however be unrealistic to assume or project a uniform transition period for every member of particular grades. This indicates ambiguity or vagueness concerning membership of the same state of the system and should better be perceived as having imprecise boundaries that facilitate gradual transition from membership to non-membership, and vice versa. In the previous works, it is introduced a method to deal with problem of lack of observations for some variable: by building of a hidden Markov model or Markov switching model, (Udom and Ebedoro, 2019) that takes into account latent sources of heterogeneity (Ugwuowo and McMclean, 2000). However, concerning hidden Markov model for manpower planning analysis, observation shows that transitions from the latent states are not free of ambiguity or vagueness. For instance, in (Udom and Ebedoro, 2019), there is no clear cut concerning the value of transition probabilities for members of (movers, mediocre and stayers) latent subclasses common to the entire personnel categories. Thus, the definition of the latent subclasses is not precise. It is subject to vagueness because the value of probability which qualifies an individual to belong to each of the distinguishable latent classes is not common to the entire personnel categories. Therefore, real applications of Markov Models in manpower planning strongly indicate the need of introducing a new method for estimating the above-mentioned probabilities, which is the prime motivating factor for considering fuzzy logic and fuzzy reasoning in non-homogeneous Markov manpower systems. Symeonaki and Kalamatianou, (2011) studied Markov systems with fuzzy states for describing students' educational progress in Greek universities. In the paper, they provided a model that projects students' transitions among progress levels as it relates to academic year.

Oczki. (2014) forecasted internal labour supply with the use of Markov chain analysis. In his paper, methods for forecasting internal labour supply were classified into qualitative, such as staffing charts, replacement charts, skill inventory etc. and quantitative, such as analysis of wastage were discussed. A classical Markov chain approach was implemented for internal labour supply analysis for a retail store, where the employees of the company were divided into (homogeneous) classes based on their job positions in the store's organizational hierarchy. Belhaj and Tkiouat (2013) modelled Heterogeneity in Manpower Planning, Dividing the Personnel System into more homogeneous Subgroups. In his work, a general framework to get a more homogeneous subgroup using Markov chain theory in Manpower planning was suggested, to improve the prediction of future behaviour. Proper Manpower planning analysis is critical for theoretical guidance in organizational budget planning, and to the management actions required to achieve desired objectives, such as devising strategies to ensuring that changes take place in a desired direction. (De Feyter and Guerry, 2016) proposed a mathematical model and algorithm for optimizing cost-effectiveness in a stochastic manpower planning system under control by recruitment. They suggested a multi-objective model that simultaneously addressed two objectives, namely minimizing the cost, and maximizing the desirability degree of the attained personnel structure. In a stochastic environment of the manpower model, internal transitions and wastage were considered as uncontrollable parameters model and were random variables. Vassiliou, (2022), studied Limiting Distributions of the Non-Homogeneous Markov System in Stochastic Environment in Continuous Time. In the paper, he stated that ordinary non-homogeneous Markov process is a very special case of an S-NHMSC. He then studied the expected population structure of S-NHMSC, the first central classical problem of finding the condition under which the asymptotic behaviour of the expected population structure exists, and second central problem of finding which expected population structures are possible limiting ones provided the limiting vector of input probabilities into the population is controlled. However, in the real world, manpower systems possess a number of imprecise and dynamic humanistic factors which play a significant role in their overall behaviours. Consequently, most of the decision making takes place in a dynamic fuzzy environment in which the goals, the constraints and the impacts of possible actions are not precisely known. The concept of a fuzzy – non homogeneous Markov system (F-NHMS) was introduced and defined for the first time in (Symeonaki *et al,* 2002). In the study, in an effort to deal with the uncertainty introduced in the estimation of the transition probabilities and the input probabilities in Markov systems, the theory of fuzzy logic and fuzzy reasoning were combined with the theory of Markov system and the concept of a fuzzy non-homogeneous Markov system was introduced. Markov chain method in calculation of personnel recruitment needs was discussed in (Setyaningrum and Abdurachman, 2022). The study used Markov chain method with an error calculation which is the result of comparison with the conditions of the number of recruits in the previous period so that error value is known and can be used in the next period using a computerized system. In their study, a way of attaining an optimal recruitment using cluster analysis technique in a fuzzy environment when an average time for completing a job is given as a fuzzy number was illustrated. In (De Feyter and Guerry, 2009), a method for evaluating recruitment policies in stochastic manpower planning based on time-homogeneous Markov theory, was presented. In the study, recruitment strategies were evaluated using fuzzy set theory, where the most preferable strategy can be chosen. In (Mutingi and Mbohwa, 2012), fuzzy system dynamics and optimization with application to manpower systems was presented. In this study, a fuzzy systems dynamics modeling approach was proposed to enable the policy maker to develop reliable dynamic policies relating recruitment, training, and available skills, from a systems perspective. In modelling manpower systems, sources of personnel differences are classified into observable and latent sources, where the latent sources could be classified into environmental factors and individual traits. A handful of papers such as (Guerry, 2011) have been devoted to partitioning personnel systems based on these latent factors, to handle their sources of personnel differences. (Guerry, 2011) discussed hidden heterogeneity in manpower Systems: A Markov-switching model approach. In this work, a two-step procedure was introduced for incorporating personnel heterogeneity into manpower modelling. Thus, for this present study, a fuzzy set theory is introduced to incorporate specific latent factors (individual traits) in the analysis of manpower systems based on the concept of non-homogeneous Markov theory. Fuzzy partitioning is introduced to classify individuals in each personnel category (determined by observable variables) into fuzzy states based on (Advance and Naive) levels of combination of a pair of specific individual traits (latent attributes).

We first consider a manpower system which is stratified into categories (states) based on the organizational attribute of interest, say grade, and let $G_1, G_2, \dots, G_k$ be the set of states that are assumed to be exhaustive and exclusive (that is, each member of the personnel system belongs to one and only one of the states at any given time). Consider a discrete time scale $t = 1,2, \dots,$ and denote the structure of the system at any given time t by the row vector, $\boldsymbol{N}(t) = [N_1(t), N_2(t), \dots, N_k(t)]$, where $N_i(t)$ is the expected number of members in grade $G_i (i = 1,2, \dots, k)$ at time t. Denote also $\{T(t)\}_{t=1}^{\infty}$ to be a sequence indicating the total number of members in the system at time t, and $\Delta T(t) = T(t+1) - T(t)$. Let $\{\boldsymbol{P}(t)\}_{t=1}^{\infty}$ be the sequence of transition probability matrices between states, $\{\boldsymbol{P_0}(t)\}_{t=1}^{\infty}$ be the sequence of vectors of probabilities of allocating new recruits to the state G_i, and

$\{\boldsymbol{P}_{\boldsymbol{k+1}}(t)\}_{t=1}^{\infty}$ be the sequence of vectors of probabilities of wastages from the grades G_i of the system. Let $\boldsymbol{Q}(t) = \boldsymbol{P}(t) + \boldsymbol{P'}_{\boldsymbol{k+1}}(t)\boldsymbol{P}_{\boldsymbol{0}}(t)$, where $(.)'$denotes the transpose of the recruitment vector, then $\boldsymbol{Q}(t)$ is a stochastic matrix known as the total transition probability and $\{\boldsymbol{Q}(t)\}_{\boldsymbol{t=1}}^{\infty}$defines what is called an embedded non-homogeneous Markov chain. The expected number of members in the various states at time t can be obtained from the following equation: $\boldsymbol{N}(t) = \boldsymbol{N}(t-1)\boldsymbol{P}(t-1) + \Delta t(t-1)\boldsymbol{P}_{\boldsymbol{0}}(t-1)$. The system described above is called a Non-Homogeneous Markov manpower system (Vassiliou, 2018).

MATERIALS AND METHODS

Assumptions

The following are the assumptions of the model.

1. Promotion of staff is done yearly, and every employee gets promoted to the next higher grade every three years after fulfilment of promotion requirements peculiar to the grade, otherwise, no promotion for that particular staff,
2. There is no double promotion. That is $p_{ij}(t) = 0,\ for\ all\ j > i+1$
3. There is no demotion. That is $p_{ij}(t) = 0,\ for\ all\ \ j \leq i-1$
4. Probability of withdrawal and probability of recruiting new members into the system are independent with respective probabilities $p_{i0}(t)$ and $p_{0j}(t)$ $\forall i, j = 1,2,\dots,k$, ie the probability of a staff leaving the system from grade i would be replaced by probability of recruitment of a new member into the system and he goes to grade j, $\left(p_{i0}(t)p_{9j}(t)\right)$
5. There is recruitment in all the grades such that $\sum_{j=1}^{`k} p_{0j}(t) = 1$
6. In the organisation under study, individual transitions behaviour between the non-fuzzy states (personnel categories) is a function of levels (naive and advance) of combination of these personnel traits.
7. The sum of the membership values for each category is one, ie $\sum_{i=1}^{k} \mu_{F_r}(i) = 1$

Notations

$G_i, i = 1,2,\dots,k$ is the grades (categories) of the system

$n_i(t)$ represents the manpower stock or number of members of staff in category G_i at time t.

$n_{ij}(t)$ is the observed flow representing number of staff in category G_i at time t who would be promoted to category G_j at time $t+1$.

$a_{ij}(t)$ denotes the probability of a member belonging to category G_i at time t, that would be promoted to G_j at time $t+1$.

$p_{i0}(t)$ represents the probability of a member leaving the system at time, $t+1$ given that he was a member of category G_i at time, $t+1$.

$p_{0j}(t)$ is the probability of allocating a recruit to category G_j at time, t.

$q_{ij}(t)$ is the total transition probability of members of the system moving from category G_i to category G_j

$p_{F_rF_s}(t)$ is the transition probability of moving from fuzzy state F_r to fuzzy state F_s at time t.

$p_{0F_s}(t)$ is the probability of recruiting a new member into the system given that he is allocated to fuzzy state F_s at time t

$p_{F_r0}(t)$ is the probability that personnel in fuzzy state F_r leaves the system at time t.

$q_{F_rF_s}(t)$ denotes total transition probability of moving from fuzzy state F_r to fuzzy state F_s

Let the aggregated manpower system of the organization be partitioned into categories based on a certain attribute of interest, say grade. Let $G_i(i = 1,2, \dots, k)$ denote the grades of the system that are assumed to be mutually exclusive and collectively exhaustive, where k is the highest of the hierarchical grades. Let G_0 a wastage category, represent external environment to which any member who leaves the system is transferred. In the analysis of differentials in manpower systems, (Ugwuowo and McClean, 2000), sources of personnel differences are classified into observable and latent sources. The latent sources of personnel differences were classified into individual traits and environmental factors (see Ugwuowo and McClean, 2000). However, for the purpose of this study, we restrict environmental factors only to organizational culture, and assume that the influence of organizational culture on individual career development is homogeneous for every member of the system. [n any organizational manpower system, individual traits are very diverse, and as such, the influence of individual traits on career development (or progress) is also very diverse for various members of the organization. In personality study, individual (personality) traits can be partitioned (Ali, 2017) into five classes, viz; Openness, Conscientiousness, Extraversion, Agreeableness, and Neuroticism. For the purpose of this work, only Openness and Conscientiousness are considered. It is assumed that individual transitions behaviour between the personnel categories are a function of the levels (naïve and advance) of combination of these personnel traits, thus, the fuzzy states for this work are partitioned based on these combinations. Fuzzy partitions are linguistic representations of their universe of discourse, (Symeonaki and Stamou, 2004) and therefore their elements are linguistic terms like 'low', medium', 'high', etc. For this work, the fuzzy partitions will be formulated in terms of 'Naive' and 'Advance' levels of the combination of the aforementioned individual traits. Therefore we consider that $F = \{F_1, F_2, F_3, F_4\}$ is the fuzzy state space of the system, where F_1describes the combination of naive level of openness and naive level of conscientiousness, F_2 describes advance level of openness and naive level of conscientiousness, F_3 describes naive level of openness and advance level of conscientiousness, while F_4 describes advance level of openness and advance level of conscientiousness.

Let $\mu_{F_r}(.): G_i \longrightarrow [0,1]$ denote the membership function of the fuzzy state $F_r, r = 1,2,3,4$. It is assumed that $F = \{F_1, F_2, F_3, F_4\}$ defines fuzzy partition on G_i such that $\sum_{r=1}^{4} \mu_{F_r}(.) = 1$. Consider the partitioning of each of the categories $G_i(i = 1,2, \dots, k)$ of a manpower system into the fuzzy states $F_r(r = 1, .., 4)$ based on the member's traits, as delineated above. Then the set $\{F_1, F_2, F_3, F_4\}$ constitutes the fuzzy events (sets) and let Π be a k x 4 matrix of the membership values for the fuzzy states, then.

$$\Pi = \begin{bmatrix} \mu_{F_1}(1) & \mu_{F_2}(1) & \mu_{F_3}(1) & \mu_{F_4}(1) \\ \mu_{F_1}(2) & \mu_{F_2}(2) & \mu_{F_3}(2) & \mu_{F_4}(2) \\ \mu_{F_1}(3) & \mu_{F_2}(3) & \mu_{F_3}(3) & \mu_{F_4}(3) \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{F_1}(k) & \mu_{F_2}(k) & \mu_{F_3}(k) & \mu_{F_4}(k) \end{bmatrix} \quad (1)$$

Definition 1: Given for example two fuzzy events, A and B, with $\mu_{F_A}(.)\ and\ \mu_{F_B}(.)$ being the membership functions of event A and event B, respectively. The product of the two fuzzy events (sets) A and B is defined by (Bhattacharyya 1998) as

$$A.B \leftrightarrow \mu_{F_{A.B}} = \mu_{F_A}.\mu_{F_B} \quad (2)$$

Definition 2: Given two fuzzy events, A and B, with $\mu_{F_A}(.)\ and\ \mu_{F_B}(.)$ being the membership functions of event A and event B, respectively. The conditional probability of fuzzy event A given a fuzzy event B is defined by (Bhattacharyya, 1998) as

$$Prob[A|B] = \frac{prob[A.B]}{Prob[B]};\ \ Prob[B] > 0 \quad (3)$$

Between-States Transition Probabilities for Non-Homogeneous Markov Fuzzy Manpower System.

Let $X_t\ and\ X_t^f$ denote non-fuzzy and fuzzy states of the manpower system at time, t respectively. For $X_{t,}$ define $n_i(t) = \sum_{j=1}^{k} n_{ij}(t)$ to represent the manpower stock or the number of personnel in category, G_i, at time, t, where the observed flow, $n_{ij}(t)$, denotes the number of personnel in category G_i at time, t that would be promoted to category $G_j (i,j = 1,2,\ldots,k)$ at time $t+1$. Define $a_{ij}(t) = prob(X_{t+1} = G_j/X_t = G_i)$ to be the probability that a member belonging to personnel grade, G_i, at time, t, would be promoted to grade G_j $(i,j = 1,2,\ldots,k)$ at time, $t+1$. For the non-homogenous Markov chain $a_{ij}(t)$, the maximum likelihood estimate can be computed as $a_{ij}(t) = \frac{\sum_t n_{ij}(t)}{\sum_t n_i(t)}$ (4)

For X_t^f, let $P(F,t)$ denote a 4 x 4 probability matrix of transitions between the fuzzy states, F_r.

$$P(F,t) = \begin{bmatrix} p_{F_1F_1}(t) & p_{F_1F_2}(t) & p_{F_1F_3}(t) & p_{F_1F_4}(t) \\ p_{F_2F_1}(t) & p_{F_2F_2}(t) & p_{F_2F_3}(t) & p_{F_2F_4}(t) \\ p_{F_3F_1}(t) & p_{F_3F_2}(t) & p_{F_3F_3}(t) & p_{F_3F_4}(t) \\ p_{F_4F_1}(t) & p_{F_4F_2}(t) & p_{F_4F_3}(t) & p_{F_4F_4}(t) \end{bmatrix} \quad (5)$$

The element, $p_{F_rF_s}(t)$, of the matrix, $P(F,t)$, represents the probability that a personnel in category, G_i, with a particular level of combination of the personnel traits at time, t, would possess or move to another level of combination at time, $t+1$. It can be calculated as follows;

$$p_{F_rF_s}(t) = Prob[X_{t+1}^f = F_s/X_t^f = F_r] = \frac{prob[X_{t+1}^f = F_s, X_t^f = F_r]}{prob[X_t^f = F_r]} \quad (6)$$

$$prob\left[X_{t+1}^f = F_s, X_t^f = F_r\right] = \sum_{i=1}^{k}\sum_{j=1}^{k} prob[X_{t+1} = G_j, X_t = G_i]\mu_{F_rF_s}(i,j),$$

$$prob\left[X_{t+1}^{f} = F_s, X_t^{f} = F_r\right] = \sum_{i=1}^{k}\sum_{j=1}^{k} prob[\,X_{t+1} = G_j/X_t = G_i]\; prob[X_t = G_i]\; \mu_{F_r}(i)\mu_{F_s}(j)$$

$= \sum_{i=1}^{k}\sum_{j=1}^{k} a_{ij}(t)\; prob[X_t = G_i]\; \mu_{F_r}(i)\mu_{F_s}(j)$ (7)

Again, $prob\left[X_t^{f} = F_r\right] = \sum_{i=1}^{k} prob[\,X_t = G_i]\mu_{F_r}(i)$ (8)

$p_{F_rF_s}(t) = \frac{\sum_{i=1}^{k}\sum_{j=1}^{k} a_{ij}(t)\; prob[X_t=G_i]\mu_{F_r}(i)\mu_{F_s}(j)}{\sum_{i=1}^{k} prob[X_t=G_i]\mu_{F_r}(i)}$ (9)

$p_{F_rF_s}(t) = (K_{F_r}(t))^{-1} \sum_{i=1}^{k}\sum_{j=1}^{k} a_{ij}(t) prob[X_t = G_i]\; \mu_{F_r}(i)\mu_{F_s}(j)$ (10)

where $K_{F_r}(t) = \sum_{i=1}^{k} prob[X_t = G_i]\mu_{F_r}(i)$.

Wastage Probabilities for Non-Homogeneous Markov Fuzzy Manpower System

Let 0 demote the external environment to which a member who leaves the system is transferred. Let $p_{i0}(t)$ be the probability that a member who leaves the system at time $t + 1$ was a member of G_i at time t. then

$p_{i0}(t) = prob[member\ leaves\ manpower\ system\ at\ time\ t + 1/X_t = G_i]$ (11)

Similarly,$P_{F_r0} = prob\left[member\ leaves\ the\ manpower\ system\ at\ time(\ t + 1)/X_t^{f} = F_r\right]$ (12)

$P_{F_r0}(t) = \frac{prob\left[member\ leaves\ the\ system\ at\ time\ (t+1), X_t^{f}=F_r\right]}{prob\left[X_t^{f}=F_r\right]}$ (13)

$$prob\left[member\ leaves\ the\ system\ at\ (t + 1), X_t^{f} = F_r\right] = \sum_{j=1}^{k} prob[member\ leaves\ at\ (t + 1),\ X_t = G_i]\mu_{F_r}(i)$$

$$= \sum_{i=1}^{k} prob[member\ leaves\ the\ system\ at\ time\ (t + 1)/\ X_t = G_i]prob[X_t = G_j]\mu_{F_r}(i)$$

prob$\left[member\ leaves\ at\ time\ t + 1, X_t^{f} = F_r\right] = \sum_{i=1}^{k} p_{i0}(t)\; prob[X_t = G_i]\mu_{F_r}(i)$ (14)

$prob\left[X_t^{f} = F_r\right] = \sum_{i=1}^{k} prob[\,X_t = G_i]\mu_{F_r}(i)$ (15)

$p_{F_r0}(t) = \frac{\sum_{i=1}^{k} p_{i0}(t) prob[X_t=G_i]\mu_{F_r}(i)}{\sum_{i=1}^{k} prob[X_t=G_i]\mu_{F_r}(i)}$ (16)

$P_{F_r0}(t) = \left(K_{F_r}(t)\right)^{-1} \sum_{i=1}^{k} p_{i0}(t)\; prob[X_t = G_i]\mu_{F_r}(i)$ (17)

Recruitment Probabilities for the Non-Homogeneous Markov Fuzzy Manpower System

Considering that the individuals (new entrants) are recruited from the external environment, 0, into the manpower system in time period, t,

Let $P_{0j}(t) = prob[X_t = G_j / new\ member\ is\ recruited\ into\ the\ system]$; (18)

Similarly, $p_{0F_s}(t) = prob[X_t^f = F_s / new\ member\ is\ recruited\ into\ the\ system]$ (19)

$$'P_{0F_s}(t) = \frac{prob[X_t^f = F_s,\ \ new\ member\ is\ recruited\ into\ the\ system]}{prob[new\ member\ is\ recruited\ into\ the\ system]} \quad (20)$$

But $prob\left[X_t^f = F_s, new\ member\ is\ recruited\ into\ the\ system\right]$

$$= \sum_{j=1}^{k} prob[\ X_t = G_j, new\ member\ is\ recruited\ into\ the\ system]\mu_{F_r}(j)$$

$= \sum_{j=1}^{k} prob[\ X_t = G_j /\ new\ member\ is\ recruited\ into\ the\ system]$x
$prob[new\ member\ is\ recruited\ into\ the\ system]\mu_{F_r}(j)$ (21)

$$p_{0F_s}(t) = \sum_{j=1}^{k} p_{0j}(t)\mu_{F_s}(j) \quad (22)$$

Total Transition Probability Matrix of the Non-Homogeneous Markov Fuzzy Manpower System

Let $Q(t) = \begin{bmatrix} q_{11}(t) & q_{12}(t) & \cdots & q_{1k}(t) \\ q_{21}(t) & q_{22}(t) & \cdots & q_{2k}(t) \\ \cdots & q_{i,j} & \cdots & \cdots \\ q_{k1}(t) & q_{k2}(t) & \cdots & q_{kk}(t) \end{bmatrix}$, where the $(i,j) -$ elements $(q_{ij}(t))$ of the matrix is given by $q_{ij}(t) = a_{ij}(t) + p_{i0}(t)p_{0j}(t)$ (23)

The $q_{ij}(t)$ expresses the total probability that either a member who is in (crisp) state, G_i, is promoted to state, G_j, $(p_{ij}(t))$, or a member that is in state, G_i, leaves the system, $(p_{i0}(t))$, and a new member is recruited and allocated to state, G_j $\left(P_{0j}(t)\right)$

$$\text{Similarly, let } Q_f(t) = \begin{bmatrix} q_{F_1F_1}(t) & q_{F_1F_2}(t) & q_{F_1F_3}(t) & q_{F_1F_4}(t) \\ q_{F_2F_1}(t) & q_{F_2F_2}(t) & q_{F_2F_3}(t) & q_{F_2F_4}(t) \\ q_{F_3F_1}(t) & q_{F_3F_2}(t) & q_{F_3F_3}(t) & q_{F_3F_4}(t) \\ q_{F_4F_1}(t) & q_{F_4F_2}(t) & q_{F_4F_3}(t) & q_{F_4F_4}(t) \end{bmatrix} \quad (24)$$

represent the total transition probability matrix of the non-homogeneous Markov manpower system with the four fuzzy states, the elements of the matrix which are given by;

$$q_{F_rF_s}(t) = p_{F_rF_s}(t) + p_{F_{r0}}(t)p_{0F_s}(t) \quad (25)$$

$$q_{F_rF_s}(t) = \left(K_{F_r}(t)\right)^{-1}\sum_{i=1}^{k}\sum_{j=1}^{k} a_{ij}(t)\, prob[X_t = G_i](\,\mu_{F_r}(i)\mu_{F_s}(j) + \left(\left(K_{F_r}(t)\right)^{-1}\sum_{i=1}^{k} p_{i0}(t)\, prob[X_t = G_i]\mu_{F_s}(i)\right)\left(\sum_{i=1}^{k} p_{0j}(t)\mu_{F_r}(j)\right)$$

$= \left(K_{F_r}(t)\right)^{-1}\sum_{i=1}^{k}\sum_{j=1}^{k}\mu_{F_r}(i)\mu_{F_s}(j)\left(a_{ij}(t) + p_{i0}(t)p_{0j}(t)\right)prob[X_t = G_i]$.

$$q_{F_rF_s}(t) = \left(K_{F_r}(t)\right)^{-1}\sum_{i=1}^{k}\sum_{j=1}^{k} q_{ij}(t) prob[X_t = G_i]\mu_{F_r}(i)\mu_{F_s}(j) \quad (26)$$

However, there are known and important connections between $Q(t)$ and $Q_f(t)$. It has been shown in (Bhattacharyya, 1968) that if Markov chain associated with the process of non-fuzzy states are irreducible, then, the corresponding Markov chain associated with the fuzzy states are also irreducible. That is, if $Q(t)$ is irreducible, then $Q_f(t)$ is also irreducible. Thus, in a similar way but extending to the case of non-homogeneous Markov manpower system where transition probabilities of members are assumed to be dependent on time, we have;

$$Q_f(t) = \omega_1(t)\Pi'\omega_2(t)Q(t)\Pi \quad (27)$$

where $\omega_1(t) = diag(\theta_{1r}(t))$, with, $\theta_{1r}(t) = \left(\sum_{i=1}^{k} prob[X_t = G_i]\mu_{F_r}(i)\right)^{-1}$ and $\omega_2(t) = diag\left(\theta_{2r(t)}\right)$ with $\theta_{2r}(t) = prob[X_t = G_i]$.

By letting the row vector $N_r(t) = [N_1(t), N_2(t), N_3(t), N_4(t)]$ represent the population structure concerning the non-homogeneous manpower system with fuzzy states at time, t, the expected population structure at time, t+1, $\left(N_r(t+1)\right)$, can be estimated by the relation

$$N_r(t+1) = Q_f(t)N_r'(t) + \Delta T(t)P_{0F}'(t) \quad (28)$$

State Probability Vector for The manpower System

Theorem 1. (Symeonaki, 2017); if $\{A\}_{t=1}^{\infty}$ is a sequence of irreducible, regular stochastic matrices, and $\lim_{t\to\infty} A(t) = A$, then the product $\prod_{i=k}^{t} A(i)$ converges to an irreducible regular stable matrix, A^*, which is $A^* = \lim_{t\to\infty} A^t$. Now, assuming that the sequence of embedded matrices,$\{Q(t)\}_{t=1}^{\infty}$, of regular and irreducible stochastic matrix for all t, and that the $\lim_{t\to\infty} Q(t) = Q$, then $Q^* = \lim_{t\to\infty} Q^t$ is a stable stochastic matrix.

As $t \to \infty$, we have, $Q^* = \lim_{t\to\infty} Q^t = \begin{bmatrix}\pi_1 & \pi_2 & \cdots & \pi_3\\ \pi_1 & \pi_2 & \cdots & \pi_3\\ \cdots & \cdots & \cdots & \cdots\\ \pi_1 & \pi_2 & \ldots & \pi_3\end{bmatrix}$ (29)

where $\pi = \pi Q$.

$p^{(t)} = prob[X_t = G_i] \Longrightarrow p^{(t)} = p^{(1)}Q(1,t) = P^{(1)}\prod_{i=1}^{t} Q(i)$

Thus, $\lim_{t\to\infty} p^{(t)} = p^{(1)}Q^* = p^*$, \

where p^* represents any row of the matrix, Q^*.

Then, $Q_f = \lim_{t\to\infty} Q_f(t) = \omega_1\Pi'\omega_2 Q\,\Pi$. (30)

and $Q_f^* = \lim_{t\to\infty} Q_f^t$ (31)

Where, ,$\omega_1 = diag(\theta_{1r})$, with, $\theta_{1r} = \left(\sum_{i=1}^{k} p_i^* \mu_{F_r}(i)\right)^{-1}$, where p_i^* are the i-elements of the vector,p^*, $\omega_2 = diag(\theta_{2r})$ with $\theta_{2r} = p_i^*$.

RESULTS

Data below are personnel flow (recruitment (R_{0j}), promotion ($G_i, i = 1, \dots, 5$), and wastage (G_6) flows) for the organisation, Satajanus Nigeria Limited, Port Harcourt from the period 2018 to 2022, where $G_1, G_2, \dots, G_5$ represents (1) Sales associates (2) departmental managers (3) section managers (4) assistant store managers (5) Store managers; and $n_i(t)$ is the total number of individual in each category at time, t.(see appendix 1). This is implemented using MATLAB.

From appendix 1, we have

$$Q = \lim_{t\to\infty} Q(t) = \begin{bmatrix} 0.6781 & 0.2654 & 0.0205 & 0.0205 & 0.0155 \\ 0.0391 & 0.6543 & 0.2839 & 0.0130 & 0.0097 \\ 0.0549 & 0.0412 & 0.6037 & 0.2865 & 0.0137 \\ 0.0429 & 0.0321 & 0.0142 & 0.6143 & 0.2964 \\ 0.0726 & 0.0544 & 0.0242 & 0.0242 & 0.8246 \end{bmatrix}$$

$$Q^* = \lim_{t\to\infty} Q^t = \begin{bmatrix} 0.1291 & 0.1851 & 0.1906 & 0.1618 & 0.3335 \\ 0.1291 & 0.1851 & 0.1906 & 0.1618 & 0.3335 \\ 0.1291 & 0.1851 & 0.1906 & 0.1618 & 0.3335 \\ 0.1291 & 0.1851 & 0.1906 & 0.1618 & 0.3335 \\ 0.1291 & 0.1851 & 0.1906 & 0.1618 & 0.3335 \end{bmatrix}$$

The matrix, Π is estimated based on the knowledge that experts possess on system under consideration. That is, the assignment of membership values to fuzzy states is based on previous studies concerning the influence of individual traits on job performance. Studies have found positively significant association between openness and conscientiousness on individual innovativeness, (Ali, 2017). It was found that people who have higher level of openness and conscientiousness are more innovative as compared to those having low levels. Thus, it is seldom does an employee who possesses advance level of openness and advance level of conscientiousness results in low levels of job performance and innovative capability or low level of career development. Using the aforementioned experts' knowledge on manpower systems behaviour, we have

$$\Pi = \begin{bmatrix} 0.7 & 0.2 & 0.1 & 0 \\ 0.5 & 0.3 & 0.2 & 0 \\ 0.1 & 0.4 & 0.4 & 0.1 \\ 0.1 & 0.2 & 0.3 & 0.4 \\ 0 & 0.1 & 0.1 & 0.8 \end{bmatrix}$$

To estimate the elements of ω_1, we first calculate θ_{1r} based on the estimated values of the elements of matrix Q^* and Π as follows,

$$\theta_{11} = \left(\sum_{i=1}^{5} p_i^* \mu_{F_1}(i)\right)^{-1} = (0.1291 * 0.7 + 0.1851 * 0.5 + \cdots + 0.3335 * 0)^{-1} = 4.5838$$

Others are similarly obtained

$$\omega_1 = \begin{bmatrix} 4.5838 & 0 & 0 & 0 \\ 0 & 4.4783 & 0 & 0 \\ 0 & 0 & 4.8063 & 0 \\ 0 & 0 & 0 & 2.8524 \end{bmatrix}$$

$$\omega_2 = \begin{bmatrix} 0.1291 & 0 & 0 & 0 & 0 \\ 0 & 0.1851 & 0 & 0 & 0 \\ 0 & 0 & 0.1906 & 0 & 0 \\ 0 & 0 & 0 & 0.1618 & 0 \\ 0 & 0 & 0 & 0 & 0.3335 \end{bmatrix}$$

$$Q_f = \lim_{t\to\infty} Q_f(t) = \begin{bmatrix} 0.4385 & 0.2724 & 0.2105 & 0.0786 \\ 0.2414 & 0.2630 & 0.2483 & 0.2473 \\ 0.1949 & 0.2531 & 0.2534 & 0.2986 \\ 0.0800 & 0.1497 & 0.1540 & 0.6163 \end{bmatrix}$$

$$Q_f^* = \lim_{t\to\infty} {Q_f}^t = \begin{bmatrix} 0.2181 & 0.2233 & 0.2080 & 0.3505 \\ 0.2181 & 0.2233 & 0.2080 & 0.3505 \\ 0.2181 & 0.2233 & 0.2080 & 0.3505 \\ 0.2181 & 0.2233 & 0.2080 & 0.3505 \end{bmatrix}$$

DISCUSSION

Q_f is the estimated transition probability matrix. $q_{F_1F_4} = 0.0786$, represents the total probability that personnel of the system who possessed naïve levels of both personnel traits, openness and conscientiousness, at time, t, would possess the traits, openness and conscientiousness, both at advanced level at time, $t + 1$ and so on. And directly associated with Q_f is the estimated matrix, Π. of the fuzzy membership function, where the membership value$\mu_{F_1}(1) = 0.7$corresponds to the degree to which the concept of employee's possession of naive level of openness and naïve level of conscientiousness (denoted by F_1) is compatible with sales associate category (denoted by G_1), The steady state probability for the fuzzy manpower system is obtained as $[0.2181, 0.2233, 0.20280, 0.3505]$. 0.2181 is the probability of remaining in fuzzy state F_1 (that is naïve level of openness and naïve level of conscientiousness). Etc. The result suggests that greater proportion of staff would possess advance level of openness and advance level of conscientiousness in the long run compared to other levels combinations of personnel traits.

CONCLUSION

In manpower planning analyses, it is assumed that every personnel belonging in a particular homogeneous group possesses homogeneous transition behaviour. However, in what looks like a scenario which clearly leads to deviation from the homogeneity assumption, most organizations based their promotion (transition) requirements on innovative capability and job performance level This results in individuals having different promotion behaviours even though belonging in the same group, since they possess different personality traits which influence their innovativeness and productivity in different ways. In order to incorporate personality traits as well as tackling the problem of ambiguity associated with gradual promotion of employees between the crisp states of a manpower system, the methodology proposed in this study is recommended. However, this paper failed to address a situation where more than two types of personality traits and more than two linguistic variables are incorporated in a fuzzy non-homogeneous Markov manpower model. This constitutes future research.

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